UNIT-1

SAMPLE SPACE

Random Experiment: An experiment whose o/p is not certain.

Sample Space: The totality of all the possible outcomes of a random experiment.

eg. tossing a coin

$S = \{H, T\}$

tossing of 2 coins

$S = \{(H,H)\ (H,T)\ (T,H)\ (T,T)\}$

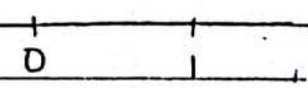
→ Sample space is determined by the purpose of the experiment

eg. If an experiment consists of examining the state of a single component - whether it is function properly (good) or it may have failed (defective)

so, sample space is

0 - defective

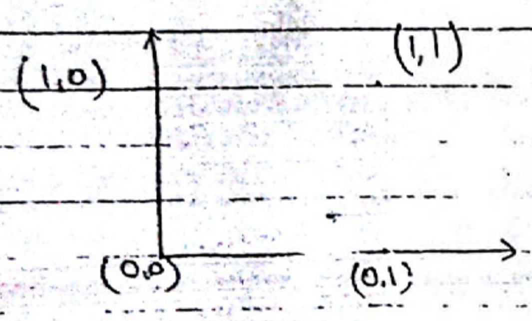
1 - good



$S = \{0, 1\}$

for 2 components

$S = \{(0,0)\ (0,1)\ (1,0)\ (1,1)\}$



2- components
experiment is to examine how many are working

0 1 2

→

Sample Space
- discrete
  - finite
  - countably infinite
- Continuous

→ finite sample space: If set of all possible outcomes of the expt is finite, then s.space if finite s.space.

→ countably infinite : No. of heads before first tail
time before first failure

→ Continuous : If the elements (points) of s.space constitute a continuum.
eg. all points on a line
all points on a line segment
all points in a plane.

Problems

1. a) RAM → 0 ROM → 1

3 chips are chosen at random

S.space = {(0,00) (001), - - . (111)}

b) 10 chips 9 - good 1 - defective

Let 1 → good 0 denotes defective

3 chips are chosen at random

S.space = {(001) (010) (100) (111)}

c) if .. then .. else statement is executed 4 times

if (C)

then {S₁}

else {S₂}

Let's then statement is denoted by 1

else " " " 0

S.space = {(0000) (0001) (0010) - - . (1111)}

EVENTS:

Events: collection of certain sample points

ie subset of sample space

Trial :- single performance of the experiment

eg. S = {(0,0) (0,1) (1,0) (1,1)}

Event = exactly one component is working

E = {(0,1) (1,0)}

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EVENT

Elementary event	Null event	Universal event
$\lvert E \rvert = 1$	$\lvert E \rvert = \phi$	$\lvert E \rvert = S$
(event consists of 1 single point)	(Impossible event)	(entire s-space is an event)

ALGEBRA OF EVENTS:

Consider an example with 5 identical tape drives.

S_0	00000	S_{21}	10101
S_1	00001	S_{22}	10110
S_2	00010	S_{23}	10111
S_3	00011	S_{24}	11000
S_4	00100	S_{25}	11001
S_5	00101	S_{26}	11010
S_6	00110	S_{27}	11011
S_7	00111	S_{28}	11100
S_8	01000	S_{29}	11101
S_9	01001	S_{30}	11110
S_{10}	01010	S_{31}	11111
S_{11}	01011		
S_{12}	01100		
S_{13}	01101		
S_{14}	01010		
S_{15}	01111		
S_{16}	10000		
S_{17}	10001		
S_{18}	10010		
S_{19}	10011		
S_{20}	10100		

Available is denoted by 1

Busy is denoted by 0

i) E_1 = atleast 4 tape drives are available
$\cancel{S} = \{S_{15}, S_{23}, S_{27}, S_{29}, S_{30}, S_{31}\}$

Complement of this event
$\bar{E_1}$ = atmost 3 tape drives are available
$= S - E_1$
$= \cancel{S_1 - S_{14}, S_{16}}$, $\{S_0$ through S_{14}, S_{16} through S_{22}, S_{24} through S_{26}, $S_{28}\}$

ii) E_2 = atmost 4 tape drives are available.
$= \{S_0$ through $S_{30}\}$

iii) $E_3 = E_1 \cap E_2$
$= \{S_{15}, S_{23}, S_{27}, S_{29}, S_{30}\}$

iv) E_4 = tape drive 1 is available.
$= \{S_{16}$ through $S_{31}\}$

v) $E_5 = E_1 \cup E_4$
$= \{S_{15}$ through $S_{31}\}$
$s \in S \mid s \in E_1$ or $s \in E_4$

MUTUALLY EXCLUSIVE EVENTS:
Events that cannot occur at same trail
$A \cap B = \phi$

COLLECTIVELY EXHAUSTIVE EVENTS:
union of all the events give you sample space
$S = \{1, 2, 3, 4, 5\}$
$S_1 = \{1, 2\}$ $S_2 = \{3, 4\}$ $S_3 = 5$
$S_1 \cup S_2 \cup S_3 = S$

$\boxed{\text{In general, } A_1 \cup A_2 \cup \ldots \cup A_n = S}$

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LAWS OF ALGEBRA:

E1. Commutative law

$A \cup B = B \cup A$ $\qquad$ $A \cap B = B \cap A$

E2. Associative law

$(A \cup B) \cup C = A \cup (B \cup C)$ $\qquad$ $(A \cap B) \cap C = A \cap (B \cap C)$

E3. Distributive law

$(A \cup B) \cap C = (A \cap C) \cup (B \cap C)$

$(A \cap B) \cup C = (A \cup C) \cap (B \cup C)$

E4. Identity law

$A \cap S = A$ $\qquad$ $A \cup \phi = A$

E5. Complementation law

$A \cup \bar{A} = S$ $\qquad$ $A \cap \bar{A} = \phi$

— Other useful relations are:

R1. Idempotent law

$A \cup A = A$ $\qquad$ $A \cap A = A$

R2. Domination law

$A \cup S = S$ $\qquad$ $A \cap \phi = \phi$

R3. Absorption law

$A \cap (A \cup B) = A$ $\qquad$ $A \cup (A \cap B) = A$

R4. De-Morgan's law

$\overline{(A \cup B)} = \bar{A} \cap \bar{B}$ $\qquad$ $\overline{(A \cap B)} = \bar{A} \cup \bar{B}$

(5)

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R5. $\overline{(\overline{A})} = A$

R6. $A \cup (\overline{A} \cap B) = A \cup B$

Problems:

1. 4 - components

S_0	0	0	0	0
S_1	0	0	0	1
S_2	0	0	1	0
S_3	0	0	1	1
S_4	0	1	0	0
S_5	0	1	0	1
S_6	0	1	1	0
S_7	0	1	1	1
S_8	1	0	0	0
S_9	1	0	0	1
S_{10}	1	0	1	0
S_{11}	1	0	1	1
S_{12}	1	1	0	0
S_{13}	1	1	0	1
S_{14}	1	1	1	0
S_{15}	1	1	1	1

0 denotes defective
1 denotes good

E_1 = all 4 components are defective
$= \{S_0\}$

E_2 = exactly 2 components are working
$= \{S_3, S_5, S_6, S_9, S_{10}, S_{12}\}$

E_3 = atmost 3 are defective
~~S_1 through S_15~~ $\{S_1$ through $S_{15}\}$

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a) $B \cap C \mid E_2 \cap E_3 = B \mid E_2$

b) $B \cup C \mid E_2 \cup E_3 = C \mid E_3$

c) $A \cup C \mid E_1 \cup E_3 = S$

d) $A \cap C \mid E_1 \cap E_3 = \phi$

2. Prove the following :-

a) $A \cup A = A$

R.H.S. $\quad A$

$A \cup \phi$

$A \cup (A \cap \bar{A})$

$(A \cup A) \cap (A \cup \bar{A})$

$(A \cup A) \cap S$

$(A \cup A) = \text{L.H.S.}$

b) $A \cup S = S$

L.H.S. $\quad A \cup S$

$A \cup (A \cup \bar{A})$

$(A \cup A) \cup \bar{A}$

$(A \cup \bar{A})$

$S = \text{R.H.S.}$

c) $A \cap \phi = \phi$

L.H.S. $\quad A \cap \phi$

$A \cap (A \cap \bar{A})$

$(A \cap A) \cap \bar{A}$

$(A \cap \bar{A})$

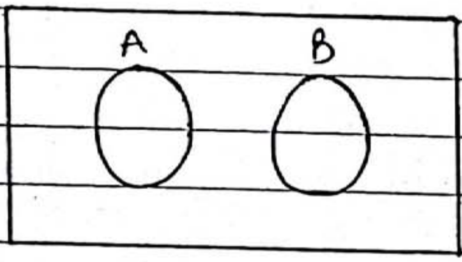
$\phi = \text{R.H.S.}$

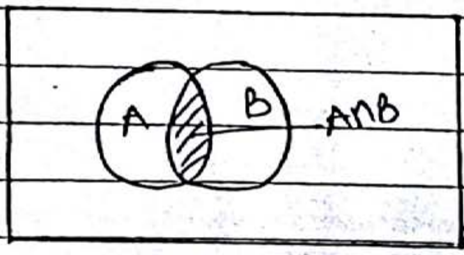
a) $A \cap (A \cup B) = A$

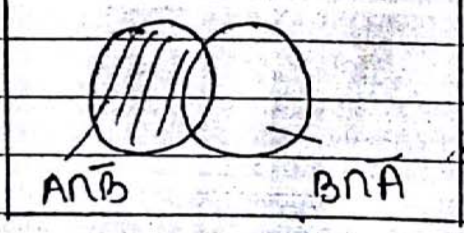
L.H.S.	R.H.S.
$A \cap (A \cup B)$	A
$(A \cup \phi) \cap (A \cup B)$	$A \cup \phi$
$A \cup (\phi \cap B)$	$A \cup (B \cap \phi)$
$A \cup \phi$	$(A \cup B) \cap (A \cup \phi)$
$A = R.H.S.$	$(A \cup B) \cap A$
	$A \cap (A \cup B) = L.H.S.$

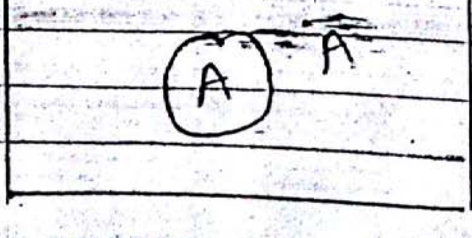
GRAPHICAL METHODS OF REPRESENTING EVENTS:

1. Venn Diagram

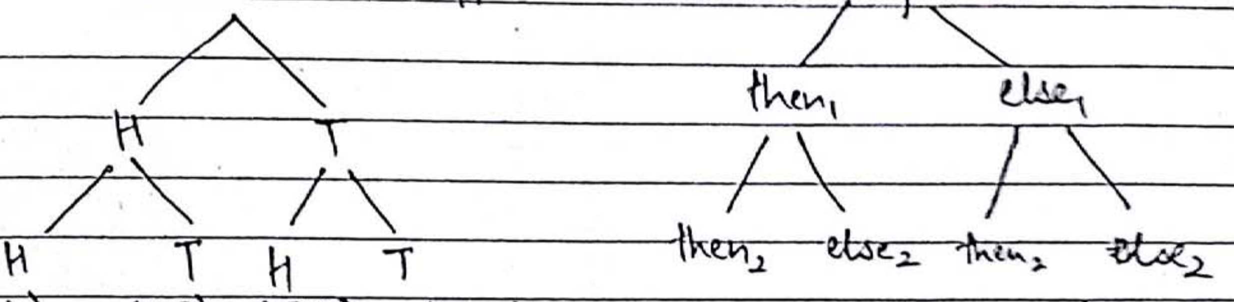








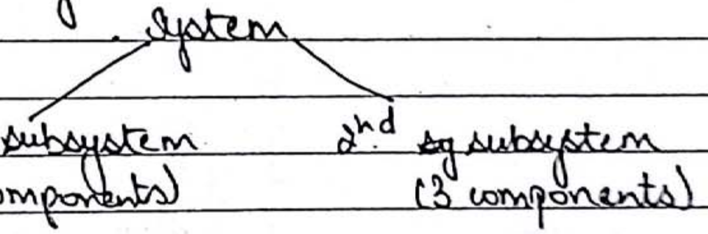
Tree Diagram



(H,H) (H,T). (T,H) (T,T)

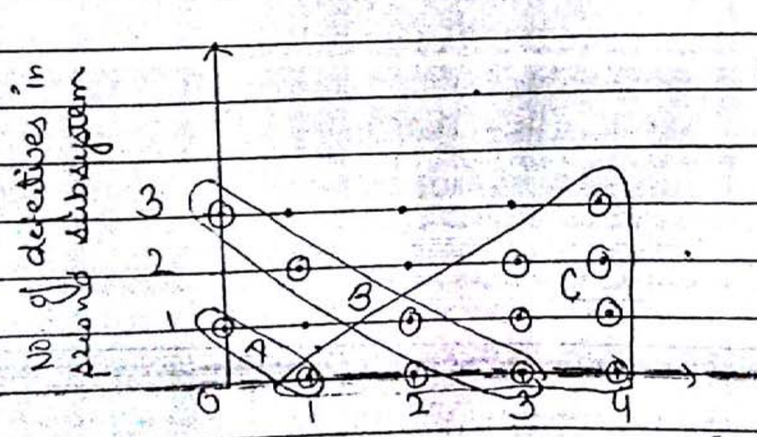
(executed 2 times).

Coordinate system



Iy IIa

$S = \{ (0,0)\ (0,1)\ (0,2)\ (0,3)$
$(1,0)\ (1,1)\ (1,2)\ (1,3)$
$(2,0)\ (2,1)\ (2,2)\ (2,3)$
$(3,0)\ (3,1)\ (3,2)\ (3,3)$
$(4,0)\ (4,1)\ (4,2)\ (4,3) \}$



Event A = "exactly 1 defective"

Event B = "exactly 3 defective components"

Event C = "1st subsystem has more defective components than 2nd subsystem"

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PROBABILITY AXIOMS:

A1. $P(A) \geq 0$; for any event A

A2. $P(S) = 1$

A3. $P(A \cup B) = P(A) + P(B)$; whenever A & B are mutually exclusive events ie $A \cap B = \phi$

Ra $P(\bar{A}) = 1 - P(A)$

proof:

$P(A \cup \bar{A}) = P(S)$

$P(A \cup \bar{A}) = 1$ — By A_2

$P(A) + P(\bar{A}) = 1$ — By A_3

$P(\bar{A}) = 1 - P(A)$

Rb $P(\phi) = 0$

Proof:

$P(\bar{S}) = 1 - P(S)$ — By Ra

$P(\bar{S}) = 1 - 1$ — By A_2

$P(\bar{S}) = 0$

$P(\phi) = 0$

Rc $P(A \cup B) = P(A) + P(B) - P(A \cap B)$

and A & B are not necessarily mutually exclusive

Proof:

$P(A \cup B) = P(A) + P(B \cap \bar{A})$ — ①

$P(B) = P(A \cap B) + P(B \cap \bar{A})$ — ②

from ②

$P(B \cap \bar{A}) = P(B) - P(A \cap B)$ — ③

put ③ in ①

$P(A \cup B) = P(A) + P(B) - P(A \cap B)$

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Qd If $A_1, A_2, \ldots, A_n$ are any events then prove that

$$P\left(\bigcup_{i=1}^{n} A_i\right) = \sum_{i=1}^{n} P(A_i) - \sum_{1\le i<j\le n} P(A_i \cap A_j) + \sum_{1\le i<j<k\le n} P(A_i \cap A_j \cap A_k) + \ldots + (-1)^{n-1} P(A_1 \cap A_2 \cap \ldots \cap A_n) \quad -(1)$$

Proof: We prove this result by mathematical induction

Let $B = (A_1 \cup A_2 \cup \ldots \cup A_{n-1})$

then

$$\bigcup_{i=1}^{n} A_i = B \cup A_n$$

Now, we apply the steps of mathematical induction

i) Basis of induction

$n=2$: for any 2 events A_1 and A_2

$$P(A_1 \cup A_2) = P(A_1) + P(A_2) - P(A_1 \cap A_2)$$

which is true

ii) Induction hypothesis : Let the above result be true for $n-1$ events

$$P(B) = P(A_1 \cup A_2 \cup \ldots \cup A_{n-1})$$

$$= \sum_{i=1}^{n-1} P(A_i) - \sum_{1\le i<j\le n-1} P(A_i \cap A_j) + \sum_{1\le i<j<k\le n-1} P(A_i \cap A_j \cap A_k) - \ldots + (-1)^{n-2} P(A_1 \cap A_2 \cap \ldots \cap A_{n-1}) \quad -(2)$$

~~Now for solving the term $B \cap A_n$~~

Now,

$$P(\underbrace{A_1 \cup A_2 \cup \ldots \cup A_{n-1}}_{B} \cup A_n) = P(B \cup A_n)$$

$$P(B \cup A_n) = P(B) + P(A_n) - P(B \cap A_n) \quad -(3)$$

for solving the term $(B \cap A_n)$ we have

$$P(B \cap A_n) = P((A_1 \cup A_2 \cup \ldots \cup A_{n-1}) \cap A_n)$$

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$$= P\left((A_1 \cap A_n) \cup (A_2 \cap A_n) \cup \cdots \cup (A_{n-1} \cap A_n)\right)$$

$$\cancel{= P\left((A_1 \cap A_n) + (A_2 \cap A_n) + \cdots + (A_{n-1} \cap A_n)\right)}$$

$$= P(A_1 \cap A_n) + P(A_2 \cap A_n) + \cdots + P(A_{n-1} \cap A_n)$$
$$- P\left((A_1 \cap A_n) \cap (A_2 \cap A_n)\right) - \cdots$$
$$+ P\left((A_1 \cap A_n) \cap (A_2 \cap A_n) \cap (A_3 \cap A_n)\right) - \cdots$$

$$\vdots$$

$$(-1)^{n-2} P\left((A_1 \cap A_n) \cap (A_2 \cap A_n) \cdots \cap (A_{n-1} \cap A_n)\right)$$

$$= P(A_1 \cap A_n) + P(A_2 \cap A_n) + \cdots + P(A_{n-1} \cap A_n) - P(A_1 \cap A_2 \cap A_n)$$
$$- P(A_1 \cap A_3 \cap A_n) - \cdots$$
$$+ P(A_1 \cap A_2 \cap A_3 \cap A_n) + \cdots$$

$$\vdots$$

$$(-1)^{n-2} P(A_1 \cap A_2 \cap A_3 \cap \cdots \cap A_{n-1} \cap A_n) \quad - (4)$$

put (ii) and (4) in (3) to get (1)

$\cancel{P(A_1 \cup A_2 \cup \cdots \cup A_{n-1} \cup A_n) = P(A_1 \cup A_2 \cup \cdots \cup A_{n-1}) + P(A_n) -}$

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⇒ How to find the probability?

1. Construct sample space
2. Assign probability
3. Identify event of interest
4. calculate the probability

example 1.1 — 32 sample points, 5 tape drives

1. Construct sample space

S_0	0 0 0 0 0	S_{21}	1 0 1 0 1
S_1	0 0 0 0 1	S_{22}	1 0 1 1 0
S_2	0 0 0 1 0	S_{23}	1 0 1 1 1
S_3	0 0 0 1 1	S_{24}	1 1 0 0 0
S_4	0 0 1 0 0	S_{25}	1 1 0 0 1
S_5	0 0 1 0 1	S_{26}	1 1 0 1 0
S_6	0 0 1 1 0	S_{27}	1 1 0 1 1
S_7	0 0 1 1 1	S_{28}	1 1 1 0 0
S_8	0 1 0 0 0	S_{29}	1 1 1 0 1
S_9	0 1 0 0 1	S_{30}	1 1 1 1 0
S_{10}	0 1 0 1 0	S_{31}	1 1 1 1 1
S_{11}	0 1 0 1 1		
S_{12}	0 1 1 0 0		
S_{13}	0 1 1 0 1		
S_{14}	0 1 1 1 0		
S_{15}	0 1 1 1 1		
S_{16}	1 0 0 0 0		
S_{17}	1 0 0 0 1		
S_{18}	1 0 0 1 0		
S_{19}	1 0 0 1 1		
S_{20}	1 0 1 0 0		

2. Assign the probability

$P(S_0) = 1/32$

$P(S_1) = 1/32$

$\vdots$

$P(S_{31}) = 1/32$

3. Identify event of interest

3 or more components are working

$E = \{S_7, S_{11}, S_{13}, S_{14}, S_{15}, S_{19}, S_{21}, S_{22}, S_{23}, S_{25}$ through $S_{31}\}$

4. Calculate probability

$$P(E) = \frac{16}{32} = \frac{1}{2}$$

Problems 1. 6 I/O buffers

A = "at least 2 but no more than 5 buffers are occupied"

$$P(A) = \frac{{}^6C_2 + {}^6C_3 + {}^6C_4 + {}^6C_5}{64}$$

$$P(A \cap B) = \frac{{}^6C_3 + {}^6C_4 + {}^6C_5}{64}$$

B = "atleast 3 but no more than 5 buffers occupied"

$$P(B) = \frac{{}^6C_3 + {}^6C_4 + {}^6C_5}{64}$$

$$P(B \cap C) = \frac{{}^6C_4}{64}$$

C = "all buffers available or an even no. of buffers occupied"

$$P(A \cap C) = \frac{{}^6C_2 + {}^6C_4}{64}$$

$$P(C) = \frac{{}^6C_6 + {}^6C_2 + {}^6C_4}{64}$$

P(atleast one of the events A, B and C occurs)

$= P(A \cup B \cup C)$

$= P(A) + P(B) + P(C) - P(A \cap B) - P(A \cap C) - P(B \cap C) + P(A \cap B \cap C)$

$= \frac{{}^6C_4}{64}$ Ans

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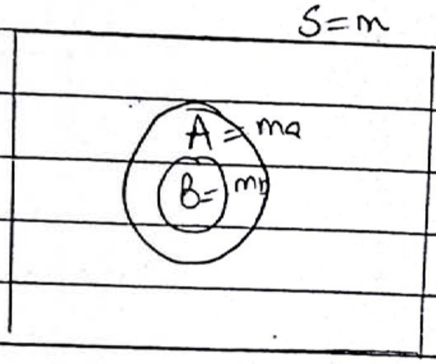
2. event B is contained in event A. Prove

Let m be no. of sample points in S.

m_A = no. of sample points in A

m_B = " " " " " B



$$P(A) = \frac{m_A}{m} \quad -\text{(I)}$$

$$P(B) = \frac{m_B}{m} \quad -\text{(II)}$$

$m_b \leq m_a$

$\frac{m_b}{m_a} \leq 1$

Dividing (II) by (I)

$$\frac{P(B)}{P(A)} = \frac{m_B}{m_A}$$

$$\frac{P(B)}{P(A)} \leq 1$$

$$\boxed{P(B) \leq P(A)}$$

Hence proved

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COMBINATORIAL PROBLEMS:

Consider the sample space $S = \{s_1, s_2, \dots s_n\}$
such that $P(s_i) = p_i$

~~($P(s_i)$: prob. of an element of S)~~

then $P(s_1) + P(s_2) + \dots + P(s_n) = 1$

$\sum_{i=1}^{n} p_i = 1$ (when events are not equally likely)

eg 1.2 If B then s_1 else s_2
exp consists of 2 successive executions of if statement

$$S = \{\underbrace{(s_1, s_1)}_{t_1}, \underbrace{(s_1, s_2)}_{t_2}, \underbrace{(s_2, s_1)}_{t_3}, \underbrace{(s_2, s_2)}_{t_4}\}$$

$$= \{t_1, t_2, t_3, t_4\}$$

$P(t_1) = 0.34$ $P(t_2) = 0.26$ $P(t_3) = 0.26$ $P(t_4) = 0.14$

E_1 = "atleast one execution of statement s_1"

$E_1 = \{(s_1, s_1)\ (s_1, s_2)\ (s_2, s_1)\}$

~~$P(E_1)$~~ $E_1 = \{t_1, t_2, t_3\}$

$$P(E_1) = P(t_1) + P(t_2) + P(t_3)$$
$$= 0.34 + 0.26 + 0.26$$
$$= 0.86$$

E_2 = "statement s_2 is executed first time"

$E_2 = \{(s_2, s_1), (s_2, s_2)\}$

$= \{t_3, t_4\}$

$$P(E_2) = P(t_3) + P(t_4)$$
$$= 0.26 + 0.14$$
$$= 0.4$$

When events are equally likely

then $P(S_i) = P_i = \dfrac{1}{n}$

If the event E consists of K equally likely sample points, then

$$P(E) = \frac{\text{no. of points in } E}{\text{no. of points in } S} = \frac{n(E)}{n(S)}$$

$$= \frac{\text{favourable outcomes}}{\text{total outcomes}}$$

$$= \frac{K}{n}$$

eg. 1.3 4 - chips

2 - good chips $\rightarrow g_1 g_2$

2 - defective chips $\rightarrow d_1 d_2$

3 chips are selected at random

~~S = { (g1, g2, d1), (g1, d1, g2) ((g1, d2, g2) (d1, d2, g1)~~

$S = \{(g_1, g_2, d_1)\ (g_1, g_2, d_2)\ (g_1, d_1, d_2)\ (g_2, d_1, d_2)\}$

E = 2 of the 3 selected chips are defectives

$P(E) = \dfrac{2}{4} = \dfrac{1}{2}$

Selection: order does not matter

$${}^nC_r = \binom{n}{r} = \frac{n!}{r!(n-r)!}$$

Arrangement: order does matter

$${}^nP_r = ({}^nP_r) = \frac{n!}{(n-r)!}$$

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Problems:

1. digits → 3, 4, 5, 6, 7

how many even 2-digit no. can be constructed?

a) repetition is allowed

T U

$^5C_1 \times {}^2C_1 = 10$

b) repetition is not allowed

T U

$^4C_1 \times {}^2C_1 = 8$

Imp.

2. 3-digit decimal no. is chosen at random find the prob. that exactly k digits are ≥ 5 for $0 \leq K \leq$

Solution

$k = 0, 1, 2, 3$

for $k=0$; no digits is ≥ 5

H T U

1-4 0-4 0-4

$n(E_1) = {}^4C_1 \times {}^5C_1 \times {}^5C_1 = 100$

for $k=1$; exactly 1 digit is ≥ 5

H T U

1-4 0-4 5-9

$^4C_1 \times {}^5C_1 \times {}^5C_1 = 100$

H T U

1-4 5-9 0-4

$^4C_1 \times {}^5C_1 \times {}^5C_1 = 100$

H T U

5-9 0-4 0-4

$^5C_1 \times {}^5C_1 \times {}^5C_1 = 125$

$n(E_2) = 325$

~~for k=2; exactly 2 digits are ≥ 5~~

H T U

1-4 5-9 5-9

$^4C_1 \times {}^5C_1 \times {}^5C_1 = 100$

H T U

5-9 5-9 0-4

$^5C_1 \times {}^5C_1 \times {}^5C_1 = 125$

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eg. 1.4

Find the prob that some randomly chosen k-digit decimal no. is a valid k-digit octal no.

Solution sample space can be chosen from the set (decimal),

$\{0, 1, 2, \ldots, 9\}$

favourable cases can be chosen from the set (octal)

$\{0, 1, 2, \ldots, 7\}$

for 3-digits

	H	T	U	
$n(S) =$	0-9	0-9	0-9	$= 10^3$

	H	T	U	
$n(E)$	0-7	0-7	0-7	$= 8^3$

$$P(E) = \frac{n(E)}{n(S)} = \frac{8^3}{10^3}$$

Similarly for k-digits $P(E) = \frac{n(E)}{n(S)} = \frac{8^k}{10^k} = \frac{4^k}{5^k}$

eg 1.5 Find the prob that a randomly chosen 3-letter sequence will not have repeated letters.

$$P(E) = \frac{26 \times 25 \times 24}{26 \times 26 \times 26}$$

eg 1.6 100 chips; 75 chips ⇒ good
25 chips ⇒ defective

12 chips are chosen at random

find the prob. that atleast one is defective

$P(\text{atleast } n) = 1 - P(\text{atmost}(n-1))$

$P(\text{atleast 1 chip is defective}) = 1 - P(\text{no chip is defective})$

$$= 1 - \frac{{}^{75}C_{12}}{{}^{100}C_{12}}$$

Date

H	T	U
5-9	0-4	5-9

$^5C_1 \times {^5C_1} \times {^5C_1} = 125$

$$n(E_3) = 350$$

for $K=3$; all digits are ≥ 5

H	T	U
5-9	5-9	5-9

$^5C_1 \times {^5C_1} \times {^5C_1} = 125$

$$n(E_4) = 125$$

Now $n(S) =$

H	T	U
1-9	0-9	0-9

$^9C_1 \times {^{10}C_1} \times {^{10}C_1} = 900$

$P(\text{exactly } K \text{ digits are } \geq 5) = \dfrac{n(E)}{n(S)}$

$= \dfrac{n(E_1) + n(E_2) + n(E_3) + n(E_4)}{n(S)}$

$= \dfrac{100 + 325 + 350 + 125}{900} = \dfrac{\cancel{900}}{\cancel{900}} = 1$ Ans

3. 15 - IC chips 5-defectives

3 chips are drawn. Find the prob that all 3 are defective.

Sol^n $n(E) = {^5C_3}$

$n(S) = {^{15}C_3}$

$P(\text{all 3 are defectives}) = \dfrac{^5C_3}{^{15}C_3}$

Date

4. Party ⇒ 5 persons
find the prob. that atleast 2 have the same birthday.

Solution: $P(\text{atleast } n) = 1 - P(\text{atmost } (n-1))$

$P(\text{atleast 2 have the same birthday}) = 1 - P(\text{no 2 persons have the same birthday})$

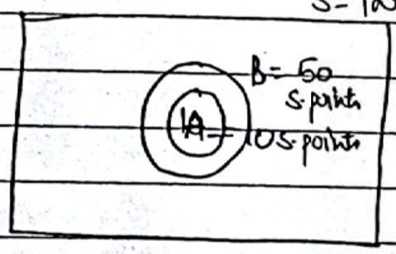
$$= 1 - \frac{365 \times 364 \times 363 \times 362 \times 361}{365 \times 365 \times 365 \times 365 \times 365}$$

CONDITIONAL PROBABILITY:

The prob. of an event A, given that event B has occured is called conditional probability.

$$P(A|B) = \frac{P(A \cap B)}{P(B)}$$

eg.



$$P(A) = \frac{10}{100}$$

$$P(A|B) = \frac{10}{50}$$

or, $$P(B|A) = \frac{P(B \cap A)}{P(A)}$$

$P(A \cap B) = P(A\|B) \cdot P(B)$	, if $P(B) \neq 0$
$= \text{or } P(B\|A) \cdot P(A)$	, if $P(A) \neq 0$
$= 0$	Otherwise

This is known as Multiplication Rule (MR)

Date

eg 1·7 box - 5000 IC chips

1000 are manufactured by company X

4000 " " " " Y

X defective chips = 10% of 1000 chips

= 100

Y defective chips = 5% of 4000 chips

= 200 chips

If a randomly chosen chip is defective, find the prob. that it is from company X.

solution Let A be the event that chip is made by company X

B " " " " " " defective

$$P(A) = \frac{1000}{5000} = \frac{1}{5}$$

$$P(B) = \frac{300}{5000} = 0.06$$

$$P(A \cap B) = \frac{100}{5000} = \frac{1}{50}$$

$$P(A/B) = \frac{P(A \cap B)}{P(B)}$$

$$= \frac{\frac{1}{50}}{0.06}$$

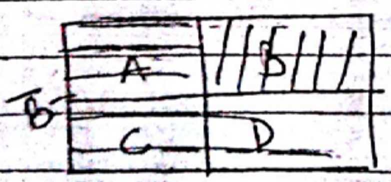
Problems: 1. 4 computer firms - A, B, C, D

Prob. of winning

$P(A) = 0.35$ $P(B) = 0.15$ $P(C) = 0.3$ $P(D) = 0.2$

Before the decision is made to award the contract, firm B withdraws its bid.

Sol:



Date

Find the new. prob of winning the bid for A, C, D

$P(A|\overline{B})$ $= \dfrac{P(A \cap \overline{B})}{P(\overline{B})}$

Prob. of A of winning the contract when B has withdrawn its bid)

$P(\overline{B}) = 1 - 0.15$

$= 0.85$

$P(A \cap \overline{B}) = P(A)$

$P(A|\overline{B}) = \dfrac{P(A)}{P(\overline{B})} = \dfrac{0.35}{0.85}$

11g

$P(C|\overline{B}) = \dfrac{P(C)}{P(\overline{B})} = \dfrac{0.3}{0.85}$

$P(D|\overline{B}) = \dfrac{P(D)}{P(\overline{B})} = \dfrac{0.2}{0.85}$

INDEPENDENCE OF EVENTS:

If the prob of an event A does not change with the occurance of another event B, then we say that event A is independent of event B.

$P(A|B) = P(A)$

$P(B|A) = P(B)$

$P(A|B) = P(A)$

$\dfrac{P(A \cap B)}{P(B)} = P(A)$

$\therefore \boxed{P(A \cap B) = P(A) \cdot P(B)}$

9/2/18

Properties:

1. If A and B are 2 mutually exclusive events, then $A \cap B = \phi$. Now, if they are independent as well, then either $P(A)=0$ or $P(B)=0$

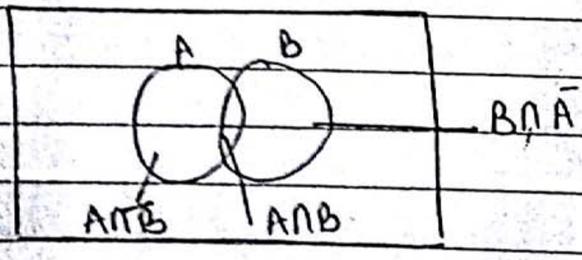
$$P(A \cap B) = P(A) \cdot P(B)$$
$$0 = P(A) \cdot P(B) \quad (\because P(A \cap B) = \phi)$$
$$P(A) = 0 \quad \text{or} \quad P(B) = 0$$

2. If an event A is independent of itself, then either $P(A)=0$ or $P(A)=1$

$$P(A \cap A) = P(A) \cdot P(A)$$
$$P(A) = P(A)^2$$

3. If events A and B are independent, events B and C are independent; then events A and C need not to be independent. ie Relation of independence is not a transitive relation.

4. If events A and B are independent, then so are $\bar{A}$ and B, B and $\bar{A}$ and $\bar{A}$ and $\bar{B}$.



$$P(B) = P(A \cap B) + P(\bar{A} \cap B)$$
$$= P(A) \cdot P(B) + P(\bar{A} \cap B).$$
$$P(B) - P(A) \cdot P(B) = P(\bar{A} \cap B)$$
$$P(B)[1 - P(A)] = P(\bar{A} \cap B)$$
$$P(B) \cdot P(\bar{A}) = P(\bar{A} \cap B)$$
$$\boxed{P(\bar{A}) \cdot P(B) = P(\bar{A} \cap B)}$$

$\parallel^{ly}$ $P(\bar{B} \cap \bar{A}) = P(\bar{B}) \cdot P(\bar{A})$

$P(\bar{B} \cap \bar{A}) = P(\bar{B}) \cdot P(\bar{A})$

eg 1.8

Date ______

CPU chips = 100 memory chips = 300

defective = 10 defective = 15

A be the event "selected CPU is defective"

B " " " "selected m/m chip is defective"

$P(A) = \frac{10}{100}$

$P(B) = \frac{15}{300}$

Since chips are selected from diff. lots, thus A and B are independent.

$$P(A \cap B) = P(A) \cdot P(B)$$
$$= \frac{10}{100} \times \frac{15}{300}$$

PAIRWISE INDEPENDENCE:

The events $A_1, A_2, \ldots, A_n$ are said to be pairwise independent iff

$$P(A_i \cap A_j) = P(A_i) \cdot P(A_j)$$

eg. 3 events A_1, A_2, A_3 are said to be pairwise independent iff

$$\left.\begin{aligned} P(A_1 \cap A_2) &= P(A_1) \cdot P(A_2) \\ P(A_2 \cap A_3) &= P(A_2) \cdot P(A_3) \\ P(A_1 \cap A_3) &= P(A_1) \cdot P(A_3) \end{aligned}\right\} \text{P.I}$$

MUTUALLY INDEPENDENT EVENTS

$P(A_1 \cap A_2 \cap A_3) = P(A_1) \cdot P(A_2) \cdot P(A_3)$ — M.I

Date

eg 1.9 Tossing 2 dice

event A = "first die results in 1, 2 or 3"

{(1,1) (1,2) (1,3) (1,4) (1,5) (1,6),
(2,1) (2,2) (2,3) (2,4) (2,5) (2,6),
(3,1) (3,2) (3,3) (3,4) (3,5) (3,6)}

event B = "first die results in 3, 4 or 5"

{(3,1) (3,2) (3,3) (3,4) (3,5) (3,6),
(4,1) (4,2) (4,3) (4,4) (4,5) (4,6),
(5,1) (5,2) (5,3) (5,4) (5,5) (5,6)}

event C = "sum of 2 faces is 9"

{(4,5) (5,4) (6,3) (3,6)}

$P(A) = \frac{18}{36} = \frac{1}{2}$

$P(B) = \frac{18}{36} = \frac{1}{2}$

$P(C) = \frac{4}{36} = \frac{1}{9}$

~~$P(A\cap B) = P(A)\cdot P(B) = \frac{1}{4}$~~

~~$P(B\cap C) = P(B)\cdot P(C) = \frac{1}{18}$~~

~~$P(A\cap C) = P(A)\cdot P(C) = \frac{1}{18}$~~

$P(A\cap B) = \frac{6}{36}\ \frac{1}{36} \neq \frac{1}{4} = P(A)\cdot P(B)$

$P(B\cap C) = \frac{3}{36}\ \frac{1}{12} \neq \frac{1}{18}\ P(B)\cdot P(C)$

$P(A\cap C) = \frac{1}{36} \neq \frac{1}{18}\ P(A)\cdot P(C)$

These events are not pairwise independent events.

$P(A\cap B\cap C) = \frac{1}{36} = P(A)\cdot P(B)\cdot P(C) = \frac{1}{36}$

These events are mutually independent events.

Eg 1.10

~~X. Tossing 2 dice~~

~~event A = "first die results in 1, 2 or 3"~~

~~$\{(1,1)\ (1,2)\ (1,3)\ (1,4)\ (1,5)\ (1,6),$~~
~~$(2,1)\ (2,2)\ (2,3)\ (2,4)\ (2,5)\ (2,6),$~~
~~$(3,1)\ (3,2)\ (3,3)\ (3,4)\ (3,5)\ (3,6)\}$~~

~~event B = "second die results in 4, 5 or 6"~~

~~$\{(4,1)\ (4,2)\ (4,3)\ (4,4)\ (4,5)\ (4,6),$~~
~~$(5,1)\ (5,2)\ (5,3)\ (5,4)\ (5,5)\ (5,6),$~~
~~$(6,1)\ (6,2)\ (6,3)\ (6,4)\ (6,5)\ (6,6)\}$~~

~~event C = "sum of 2 faces is 7"~~

~~$\{(4,3)\ (3,4)\ (5,2)\ (2,5)\ (1,6)\ (6,1)\}$~~

~~[illegible]~~ $P(A) = \cancel{\frac{18}{36}}$

Eg 1.10 Tossing 2 dice

event A = "first dice results in 1, 2 or 3"

$\{(1,1)\ (1,2)\ (1,3)\ (1,4)\ (1,5)\ (1,6),$
$(2,1)\ (2,2)\ (2,3)\ (2,4)\ (2,5)\ (2,6),$
$(3,1)\ (3,2)\ (3,3)\ (3,4)\ (3,5)\ (3,6)\}$

event B = "second die results in 4, 5 or 6"

$\{(1,4)\ (2,4)\ (3,4)\ (4,4)\ (5,6)\ (6,6),$
$(1,5)\ (2,5)\ (3,5)\ (4,5)\ (5,5)\ (5,6),$
$(1,6)\ (2,6)\ (3,6)\ (4,6)\ (5,6)\ (6,6)\}$

event C = "sum of 2 faces is 7"

$= \{(4,3)\ (3,4)\ (6,1)\ (1,6)\ (5,2)\ (2,5)\}$

$P(A) = \frac{18}{36}\;\frac{1}{2}$ $\quad P(B) = \frac{18}{36}\;\frac{1}{2}$ $\quad P(C) = \frac{6}{36}\;\frac{1}{6}$

$P(A \cap B) = \frac{9}{36}\;\frac{1}{4} = \frac{1}{4} = P(A) \cdot P(B)$

$P(B \cap C) = \frac{3}{36}\;\frac{1}{12} = \frac{1}{12} = P(B) \cdot P(C)$

$P(A \cap C) = \frac{3}{36}\;\frac{1}{12} = \frac{1}{12} = P(A) \cdot P(C)$

These events are pairwise Independent events.

$P(A \cap B \cap C) = \frac{3}{36}\;\frac{1}{12} \neq \frac{1}{24} = P(A) \cdot P(B) \cdot P(C)$

These events are not mutually independent events.

RELIABILITY:

- The reliability of a component is defined as the probability of its functioning properly.
- Let A be the event denoting that the component is functioning properly. Then its reliability is

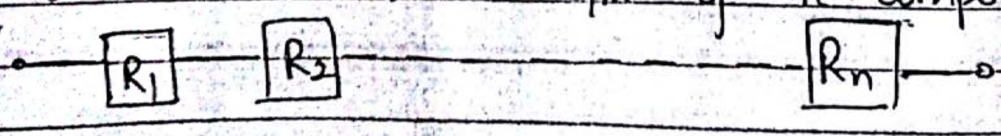
$$R = P(A).$$

Probability at working $(R_i + F_i) = 1$

→ Series System

A series system is an arrangement of components in a linear fashion such that if any of the components fails, ~~then the entire s/m fails.~~

Consider a series s/m of n components:



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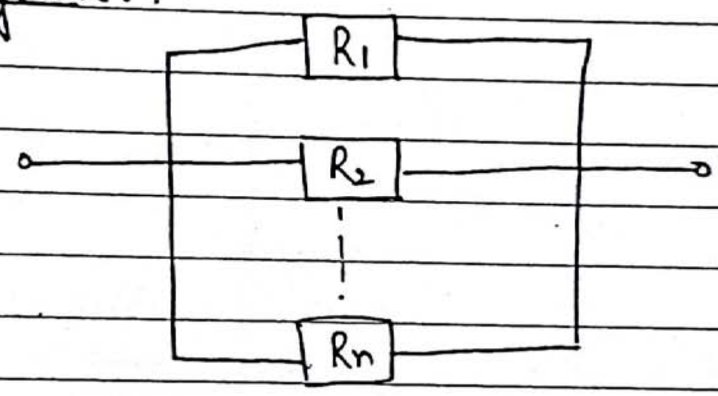
$R_s = P$ ("series s/m is working correctly")

$= R_1 \cdot R_2 \cdot \ldots \cdot R_n$

$= \prod_{i=1}^{n} R_i \quad = \underline{\underline{R^n}}$

Product law of Reliability

→ Parallel system:



A parallel system is an arrangement of components in such a way that the s/m is functioning properly if any of the components is functioning properly, i.e. the s/m will fail only when all the components fail.

$F_{(parallel)} = (1-R_1)(1-R_2) \ldots (1-R_n)$

$= \prod_{i=1}^{n} (1-R_i)$

We know $R_p + F_p = 1$

$R_p = 1 - F_p$

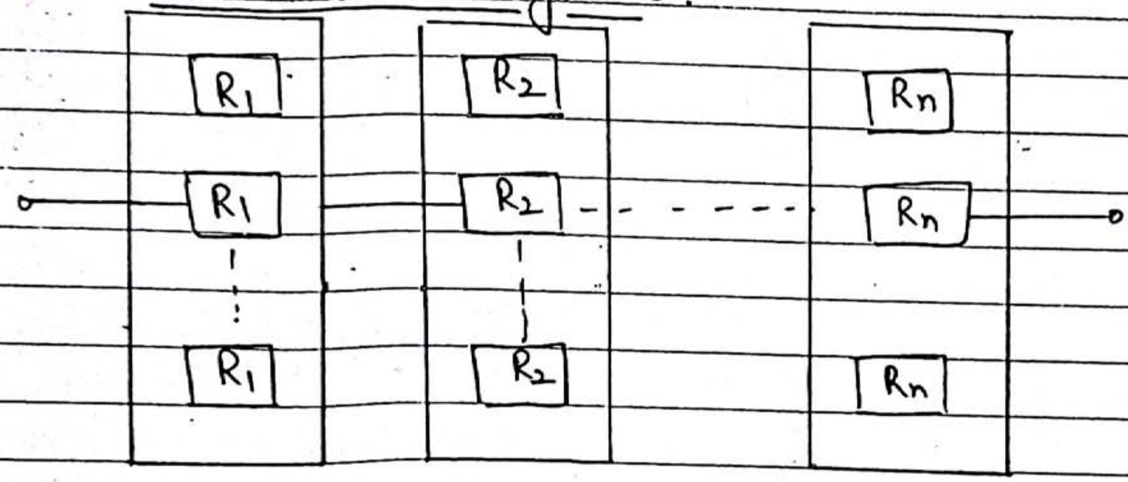
$R_p = 1 - \prod_{i=1}^{n} (1-R_i)$

$= 1 - (1-R)^n$

Product law of un-reliabilities

Date

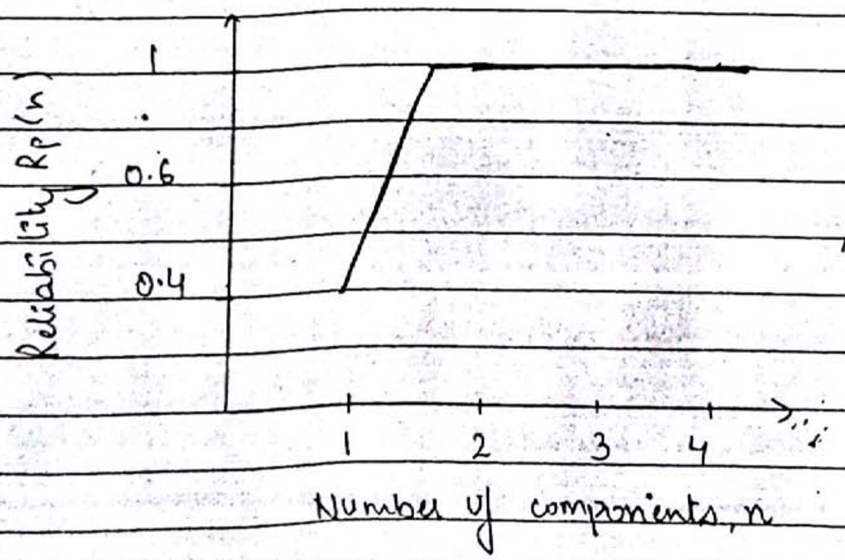
→ Series-Parallel System:



$$R_{SP} = \left[1-(1-R_1)^{n_1}\right] \cdot \left[1-(1-R_2)^{n_2}\right] \cdots \left[1-(1-R_n)^{n_n}\right]$$

$$= \prod_{i=1}^{n} \left[1-(1-R_i)^{n_i}\right]$$

→ LAW OF DIMINISHING RETURNS:



The rate of increase in reliability with each additional component decreases rapidly as n increases.

Date

Eg. 1.1

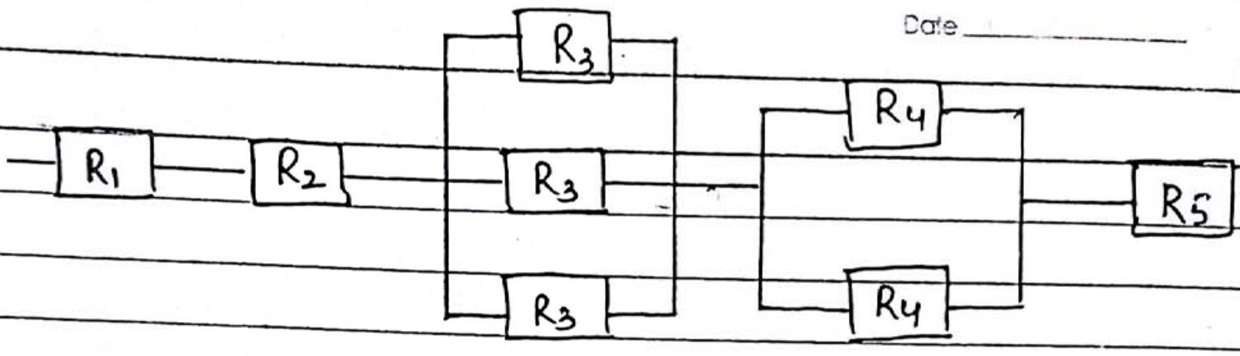


fig. Series-Parallel System

$R_1 = 0.95 \qquad R_2 = 0.99 \qquad R_3 = 0.70 \qquad R_4 = 0.75 \qquad R_5 = 0.9$

$$R_{sp} = R_1 \cdot R_2 \cdot \left[1-(1-R_3)^3\right] \cdot \left[1-(1-R_4)^2\right] \cdot R_5$$
$$= 0.95 \times 0.99 \left[1-0.3^3\right]\left[1-0.25^2\right] \cdot 0.9$$
$$= 0.772$$

Problems 1.

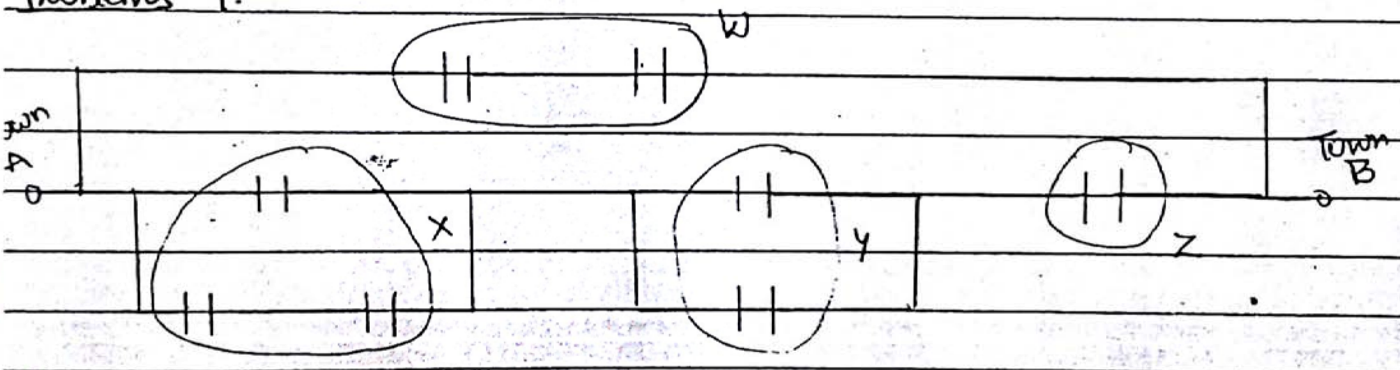


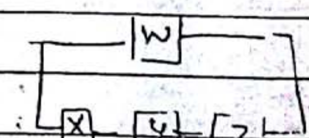
fig. a N/w of communication channels

$R(X) = \left[1-(1-R)(1-R^2)\right]$

$R(Y) = \left[1-(1-R)^2\right]$

$R(Z) = R$

$R(W) = R.R = R^2$



$R_s = 1 - [1 - R(X) \cdot R(Y) \cdot R(Z)][1 - R(W)]$

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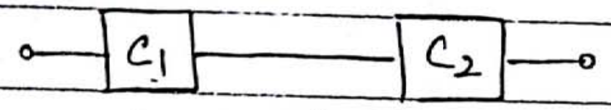
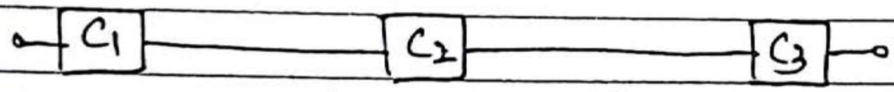
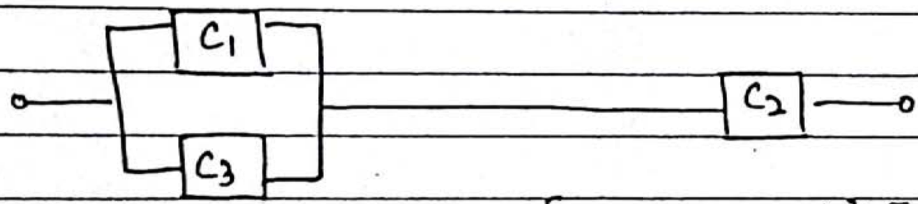
2.



$R_s = R(C_1) \cdot R(C_2)$



$R_s = R(C_1) \cdot R(C_2) \cdot R(C_3)$



$R_{sp} = \left[1 - (1 - R_{C_1,C_3})^2\right] \cdot R_{C_2}$

3. Determine the conditions under which an event A is independent of its subset B.

$$P(A \cap B) = P(A) \cdot P(B) \quad \text{— (I)}$$
$$P(A \cap B) = P(B) \quad \text{— (II)}$$
$$P(B) = P(A) \cdot P(B)$$
$$P(A) \cdot P(B) - P(B) = 0$$
$$P(B)\,[P(A) - 1] = 0$$

either $P(A) = 1$ or $P(B) = 0$

Date

General Multiplication Rule: (GMR):

Given a list of events $A_1, A_2, \ldots, A_n$ (not necessarily independent), show that

$$P(A_1 \cap A_2 \cap \cdots \cap A_n) = P[A_1 | (A_2 \cap A_3 \cap \cdots \cap A_n)] \cdot P[A_2 | (A_3 \cap A_4 \cap \cdots \cap A_n)] \cdot P[A_3 | (A_4 \cap A_5 \cap \cdots \cap A_n)] \cdots P[A_{n-1} | A_n] \cdot P(A_n)$$

Proof :- $n = 2$ events, for 2 events A_1 & A_2

acc. to conditional probability

$$P(A_1 \cap A_2) = P(A_1) \cdot P(A_2)$$

for $n-1$ events, let $A = A_1$ and $B = A_2 \cap A_3 \cap \cdots \cap A_n$

$$P(B) = P(A_2 \cap A_3 \cap \cdots \cap A_n) = P(A_2 | A_3 \cap A_4 \cdots \cap A_n) \cdot P(A_3 | A_4 \cap A_5 \cap \cdots \cap A_n)$$

$$\vdots$$

$$P(A_{n-1} | A_n) \cdot P(A_n)$$

for n events

$$P(A_1 \cap A_2 \cap \cdots \cap A_n) = P(A_1 \cap B) \quad (\because B = A_2 \cap A_3 \cap \cdots \cap A_n)$$

$$= P(A_1 | B) \cdot P(B)$$

$$= P(A_1 | A_2 \cap A_3 \cap \cdots \cap A_n) \cdot P(A_2 | A_3 \cap A_4 \cap \cdots \cap A_n)$$

$$\vdots$$

$$P(A_{n-1} | A_n) \cdot P(A_n).$$

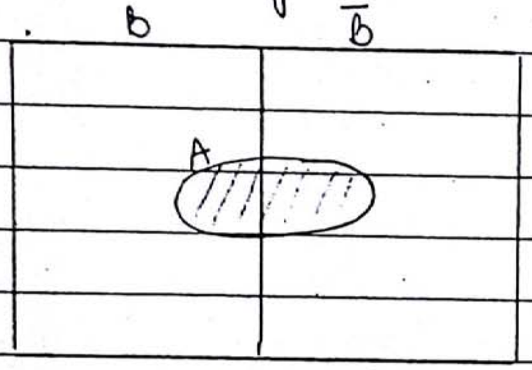
Hence proved.

Date

BAYE'S RULE:

- The Baye's Rule helps to connect the CONDITIONAL PROBABILITY $P(E_i/A)$ of E_i when A has already occured with the CONDITIONAL PROBABILITY $P(A|E_i)$ of A when E_i has already occured for each i.

→



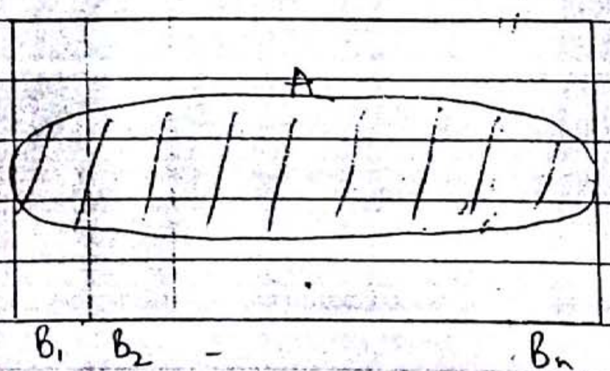
A given event B of prob. $P(B)$ partitions the sample space S into 2 disjoint subsets B and $\overline{B}$.

$$S = \{B, \overline{B}\}$$

So, $P(A) = P(A \cap B) + P(A \cap \overline{B})$

$$= P(A|B) \cdot P(B) + P(A|\overline{B}) \cdot P(\overline{B})$$

→ Similarly, If Sample Space is partitioned into n disjoint subsets then



$$P(A) = P(A|B_1) \cdot P(B_1) + P(A|B_2) \cdot P(B_2) + \cdots + P(A|B_n) \cdot P(B_n)$$

$$= \sum_{i=1}^{n} P(A|B_i) \cdot P(B_i)$$

This relation is also known as theorem of total probability.

Date

Eg 1.13 $P(R_0|T_0) = 0.94$ $P(R_1|T_0) = 0.06$

$P(R_1|T_1) = 0.91$ $P(R_0|T_1) = 0.09$

$P(T_0) = 0.45$ $P(T_1) = 0.55$

Determine

1. Prob. that 1 is received

$$P(R_1) = P(R_1|T_0) \cdot P(T_0) + P(R_1|T_1) \cdot P(T_1)$$

2. Prob. that 0 is received

$$P(R_0) = P(R_0|T_0) \cdot P(T_0) + P(R_0|T_1) \cdot P(T_1)$$

3. Prob. that 1 was transmitted, given that 1 was received

$$P(T_1|R_1) = \frac{P(R_1|T_1) \cdot P(T_1)}{P(R_1|T_1) \cdot P(T_1) + P(R_1|T_0) \cdot P(T_0)}$$

4. Prob. that 0 was transmitted, given that 0 was received

$$P(T_0|R_0) = \frac{P(R_0|T_0) \cdot P(T_0)}{P(R_0|T_0) \cdot P(T_0) + P(R_0|T_1) \cdot P(T_1)}$$

5. Prob. of an error

$$P(\text{error}) = P(R_1|T_0) \cdot P(T_0) + P(R_0|T_1) \cdot P(T_1)$$

Eg 1.14 $P(CD) = 0.02$ (Prob that chip is defective) $P(CG) = 0.98$ (Prob that chip is good)

$P(TSG|CG) = 0.95$ $P(TSD|CG) = 0.05$

$P(TSD|CD) = 0.94$ $P(TSG|CD) = 0.06$

If tested device is indicated to be defective, what is the prob. that it is actually defective

$$P(CD|TSD) = \frac{P(TSD|CD) \cdot P(CD)}{P(TSD|CD) \cdot P(CD) + P(TSD|CG) \cdot P(CG)}$$

Date

ng..s Problems:

3. $P(TD) = 0.06$ $\quad P(TG) = 0.94$

$P(TSD|TD) = 1$ $\quad P(PSG|TD) = 0$

$P(TSD|TG) = 0.02$ $\quad P(TSG|TG) = 0.98$

A randomly chosen transistor is declared defective by the tester, find the prob. that it is actually defective.

$$P(TD|TSD) = \frac{P(TSD|TD) \cdot P(TD)}{P(TSD|TD) \cdot P(TD) + P(TSD|TG) \cdot P(TG)}$$

4. $P(A) = 0.35$ $\quad P(B) = 0.15$ $\quad P(C) = 0.50$

$P(ND|A) = 0.75$ $\quad P(ND|B) = 0.95$ $\quad P(ND|C) = 0.85$

A customer receives a defective what is the prob. that it came from plant C?

$P(D|A) = 0.25$ $\quad P(D|B) = 0.05$ $\quad P(D|C) = 0.15$

$$P(C|D) = \frac{P(D|C) \cdot P(C)}{P(D|C) \cdot P(C) + P(D|A) \cdot P(A) + P(D|B) \cdot P(B)}$$

~~5.~~

Date

5.

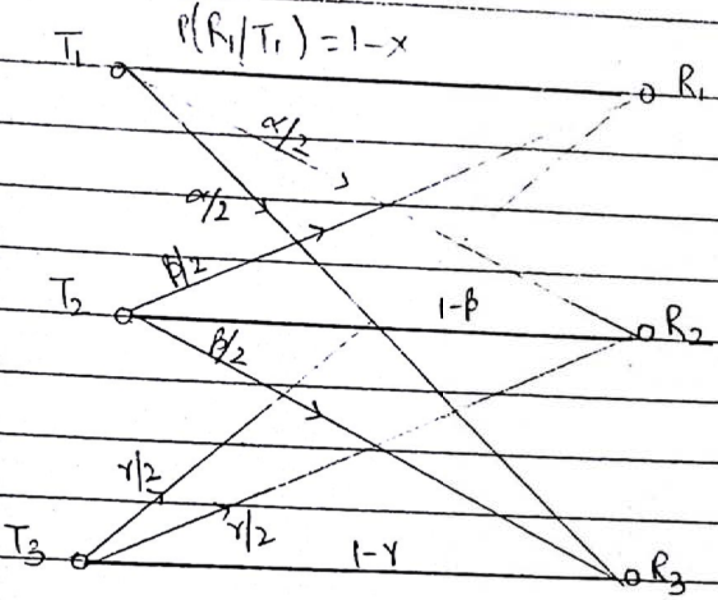


fig. a trinary communication channel : channel diagram

$P(R_1|T_1) = 1-\alpha$ $\quad P(R_2|T_1) = \alpha/2$ $\quad P(R_3|T_1) = \alpha/2$

$P(R_2|T_2) = 1-\beta$ $\quad P(R_1|T_2) = \beta/2$ $\quad P(R_3|T_2) = \beta/2$

$P(R_3|T_3) = 1-\gamma$ $\quad P(R_1|T_3) = \gamma/2$ $\quad P(R_2|T_3) = \gamma/2$

$P(T_3) = 3P(T_1)$

$P(T_2) = 2P(T_1)$

$P(T_1) + P(T_2) + P(T_3) = 1$

$P(T_1) + 2P(T_1) + 3P(T_1) = 1$

$6 \cdot P(T_1) = 1$

$P(T_1) = 1/6$

$P(T_2) = 1/3$ $\qquad P(T_3) = 1/2$

1 was received, find the prob. that 1 was sent.

$$P(T_1|R_1) = \frac{P(R_1|T_1) \cdot P(T_1)}{P(R_1|T_1) \cdot P(T_1) + P(R_1|T_2) \cdot P(T_2) + P(R_1|T_3) \cdot P(T_3)}$$

Date

$$P(\text{error}) = P(R_2|T_1)\cdot P(T_1) + P(R_3|T_1)\cdot P(T_1) + P(R_1|T_2)\cdot P(T_2) + P(R_3|T_2)\cdot P(T_2) + P(R_1|T_3)\cdot P(T_3) + P(R_2|T_3)\cdot P(T_3)$$

6. $P(W) = 0.70$ $\quad$ $P(M) = 0.30$

$P(S|W) = 0.20$ $\quad$ $P(S|M) = 0.25$

Find the prob. that a randomly selected student is

a) a woman who smokes?

$P(S \cap W) = P(S|W)\cdot P(W)$

b) a man who smokes?

$P(S \cap M) = P(S|M)\cdot P(M)$

c) A smoker?

$P(S) = P(S|W)\cdot P(W) + P(S|M)\cdot P(M)$

Date

BERNOULLI TRIALS

- A single performance of a random experiment having 2 possible outcomes ie "success" and "failure" is called a Bernoulli trial.

- The probability of success is denoted by p
- " " " failure " " " q

where, $p+q=1$.

- Let 0 denote failure and 1 denote success.

Let S_n be the sample space of an experiment involving n Bernoulli trials, defined by

$$S_1 = \{0,1\}$$

$$S_2 = \{(0,0), (0,1), (1,0), (1,1)\}$$

$$\vdots$$

$$S_n = \{2^n \text{ n-tuples of 0's and 1's}\}$$

eg.

000 - p^0q^3

001 - p^1q^2

010 - p^1q^2

011 - p^2q^1

100 - p^1q^2

101 - p^2q^1

110 - p^2q

111 - p^3q^0

$$\text{S.point} = (\underbrace{1111\cdots1}_{k\text{ 1's}}\ \underbrace{00\cdots0}_{n-k\text{ 0's}})$$

- If there are $\binom{n}{k}$ sample points, the prob of obtaining EXACTLY k successes in n trials is

$$P(k) = \binom{n}{k} p^k q^{n-k}, \quad k = 0,1,\ldots,n$$

or $P(k) = \binom{n}{n-k} p^k q^{n-k}$

Date ____________

$$\sum_{k=0}^{n} \binom{n}{k} p^k \cdot q^{n-k} = \binom{n}{0} p^0 \cdot q^n + \binom{n}{1} p^1 q^{n-1} + \cdots$$

$$= (p+q)^n = 1$$

$$\because (a+b)^n = \binom{n}{0} a^0 b^n + \binom{n}{1} a^1 \cdot b^{n-1} + \cdots + \binom{n}{n} a^n b^0$$

sample 1.16

$R_{m/n}$ = P (atleast m are working correctly)

$$= \sum_{k=m}^{n} \binom{n}{k} p^k \cdot q^{n-k}$$

$$= \sum_{k=m}^{n} \binom{n}{k} R^k \cdot (1-R)^{n-k}$$ ($\sum_{k=m}^{n}$ P(exactly m are working correctly))

$$= \sum_{k=m}^{n} P(k)$$

Now ; $R_{1/n} = \sum_{k=1}^{n} \binom{n}{k} R^k (1-R)^{n-k}$

$$= \sum_{k=0}^{n} \binom{n}{k} R^k (1-R)^{n-k} - \binom{n}{0} R^0 (1-R)^n$$

$$= (R + 1 - R)^n - (1-R)^n$$

$$= 1 - (1-R)^n$$

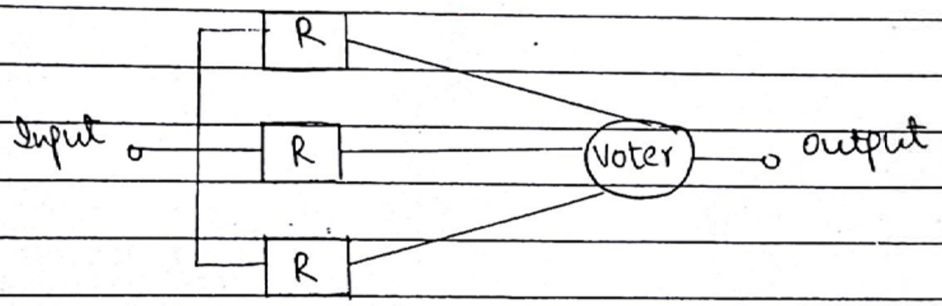
also, $R_{n/n} = \sum_{k=n}^{n} \binom{n}{k} p^{k} q^{n-k}$

$$= \sum_{k=n}^{n} \binom{n}{k} R^{k} \cdot (1-R)^{n-k}$$

$$= \binom{n}{n} R^{n} (1-R)^{0}$$

$$= R^{n}$$

TMR (TRIPLE MODULAR REDUNDANCY) SYSTEM:



A TMR is a system with 3 components, out of which atleast 2 components should function properly for correct functioning of the system.

$R_{m/n} = R_{TMR} = \sum_{k=2}^{3} \binom{3}{k} R^{k} (1-R)^{3-k}$, here $m=2$, $n=3$

$$R_{2/3} = \binom{3}{2} R^{2} (1-R)^{1} + \binom{3}{3} R^{3} (1-R)^{0}$$

$$= 3R^{2}(1-R) + R^{3}$$

$$= 3R^{2} - 3R^{3} + R^{3}$$

$$= 3R^{2} - 2R^{3}$$

Special case:

i) when $R = 1/2$, $R_{TMR} = R$

ii) when $R > 1/2$, $R_{TMR} > R$

iii) when $R < 1/2$, $R_{TMR} < R$.

eg 1.17 n-bits

prob. of successful transmission of a single bit is p

prob. of an error, $q = 1-p$

$$P(\text{successful word transmission}) = \sum_{k=0}^{e} (1-p)^k \cdot p^{n-k}$$

GENERALIZED BERNOULLI TRIALS:

cases

30 – 1 – P_1

20 – 2 – P_2

15 – 3 – P_3

35 – 4 – P_4

$$P_1 + P_2 + \cdots + P_k = 1$$

$$n_1 + n_2 + \cdots + n_k = 1$$

$$\frac{100!}{30!\,20!\,15!\,35!} P_1^{30} \cdot P_2^{20} \cdot P_3^{15} \cdot P_4^{35}$$

In general

$$\frac{n!}{n_1!\,n_2!\,n_3! \cdots n_k!} P_1^{n_1} \cdot P_2^{n_2} \cdot \cdots \cdot P_n^{n_k}$$

Date

eg 1.18 100 jobs

50 jobs - class 1 $\quad P(I) = 0.50$

30 jobs - class 2 $\quad P(II) = 0.30$

20 jobs - class 3 $\quad P(III) = 0.20$

a sample of 30 jobs is taken with replacement. Find the prob. that

1. the sample will contain 10 jobs of each class

$$\frac{30!}{10!\,10!\,10!}(0.50)^{10}(0.30)^{10}(0.20)^{10}$$

2. there will be exactly 12 jobs of class 2

$$\frac{30!}{12!\,18!}(0.30)^{12}(0.70)^{18}$$

example 1.19:

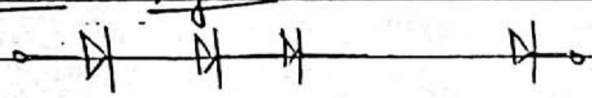
Let p_1 = diode is functioning correctly
p_2 = diode is short circuit
p_3 = diode is open circuit

Let n_1 diodes be functioning properly
n_2 diodes be short circuit
n_3 diodes be open-circuit

$$p_1 + p_2 + p_3 = 1 \quad \text{and} \quad n_1 + n_2 + n_3 = 1$$

$$\frac{n!}{n_1!\, n_2!\, n_3!} \quad p_1^{n_1} \cdot p_2^{n_2} \cdot p_3^{n_3}$$

1. Series system



Let S_1 denotes the prob. of correct functioning

$$n_3 = 0$$

$$S_1 = \sum_{\substack{n_1 \geq 1 \\ n_2 \geq 0 \\ n_1 + n_2 = n}} \frac{n!}{n_1!\, n_2!\, 0!} \quad p_1^{n_1} \cdot p_2^{n_2} \cdot p_3^{0}$$

$$= \sum_{\substack{n_1 \geq 1 \\ n_2 \geq 0 \\ n_1 + n_2 = n}} \frac{n!}{n_1!\, (n-n_1)!} \quad p_1^{n_1} \cdot p_2^{n-n_1}$$

$$= \sum_{\substack{n_1 \geq 1 \\ n_2 \geq 0 \\ n_1 + n_2 = n}} \binom{n}{n_1} p_1^{n_1} \cdot p_2^{n-n_1}$$

$$= \sum_{\substack{n_1=0 \\ n_1+n_2=n}} \binom{n}{n_1} p_1^{n_1} \cdot p_2^{n-n_1} - \binom{n}{0} p_1^0 \cdot p_2^n$$

$$= (p_1+p_2)^n - p_2^n$$

$$= (1-p_3)^n - p_2^n$$

Let s_2 denotes the prob. that the ~~diode~~ sym is short ckt.

$S_2 =$ P(all diodes are shorted)

$n_2 = n$

$$= \frac{n!}{0!\, n!\, 0!} p_1^0 p_2^n p_3^0$$

$$= p_2^n$$

Let s_3 denotes the prob. that the sym is open ckt.

$s_3 =$ P(atleast one diode is open-circuit)

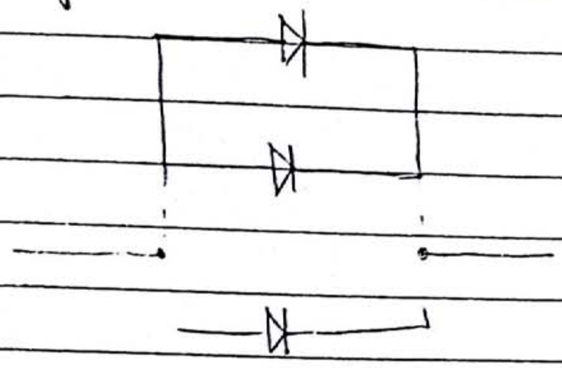
$= 1 -$ P(no diode is open ckt)

$$= 1 - \sum_{n_1+n_2}^{n} \frac{n!}{n_1!\, n_2!} p_1^{n_1} \cdot p_2^{n_2}$$

$$= 1 - (p_1+p_2)^n$$

$$= 1 - (1-p_3)^n$$

Parallel System



Let P_1 denotes the prob. of correct functioning

$$= \sum_{\substack{n_1 \geq 0 \\ n_2 = 0 \\ n_1 + n_3 = n}} \frac{n!}{n_1!\, 0!\, n_3!} P_1^{n_1} \cdot P_3^{n_3} \quad (n_2 = 0)$$

$$= (1-P_2)^n - P_3^n$$

Let P_2 denotes the sym is short ckt.

P_2 = (atleast one diode is short ckt.)

= 1 − (no diode is short ckt)

$$= 1 - \frac{n!}{n_1!\, n_3!} P_1^{n_1} \cdot P_3^{n_3}$$

$$= 1 - \frac{n!}{n_1!\,(n-n_1)!} P_1^{n_1} \cdot P_3^{n-n_1}$$

$$= 1 - (P_1 + P_3)^n$$

$$= 1 - (1-P_2)^n$$

Problems

1) if B then repeat S_1 until B_1
else repeat S_2 until B_2

$P(B = \text{true}) = p$ $P(B_1 = \text{true}) = 3/5$ $P(\bar{B}_1 = \text{false}) = 1 - \frac{3}{5} = \frac{2}{5}$

$P(\bar{B}) = 1-p$ $P(B_2 = \text{true}) = 2/5$ $P(\bar{B}_2 = \text{false}) = 1 - \frac{2}{5} = \frac{3}{5}$

exactly 1 statement is common to statements S_1 and S_2 : Write ('good day')

Prob. of printing exactly 3 'good day' messages is $\frac{3}{25}$.

Derive value of p.

Sol^n

$$\frac{3}{25} = P(B) \times P(B_1) \times P(B_1) \times P(\bar{B}_1) + P(B) \times P(B_2) \times P(B_2) \times P(\bar{B}_2)$$

$$\frac{3}{25} = p \times \frac{3}{5} \times \frac{3}{5} \times \frac{2}{5} + (1-p) \times \frac{2}{5} \times \frac{2}{5} \times \frac{3}{5}$$

$$\frac{3}{25} = \frac{18p}{125} + (1-p)\frac{12}{125}$$

$$3 = \frac{18p}{5} + \frac{(1-p)12}{5}$$

$$15 = 18p + 12 - 12p$$

$$15 = 6p + 12$$

$\cancel{5 = 2p + 4}$ $3 = 6p$

$\boxed{\cancel{p = 0.5}}$ $\boxed{p = 0.5}$

2) $P(E) = 0.0008$

Compute the prob of error in transmitting a block of 1024 bits.

Sol^n $P(\bar{E}) = 1 - 0.0008 = 0.9992$

$P(\text{Error}) = 1 - P(\text{No error})$

$$P(\text{error}) = 1 - \binom{1024}{0} (0.0008)^0 (0.9992)^{1024}$$

Let P_3 denotes the prob. that the s/m is open ckt.

$P_3 = P$ (all diodes open ckt)

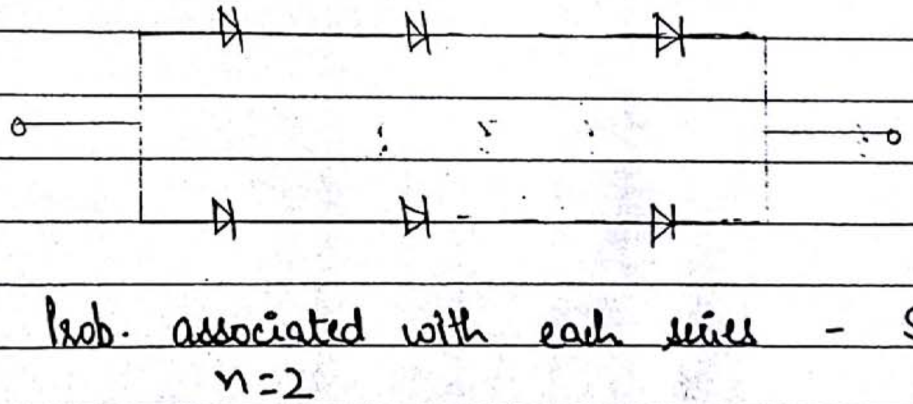
$n_3 = n$

$= \frac{n!}{n_3!} P_3^{n_3}$

$= \frac{n!}{n!} \cdot P_3^{n}$

$= P_3^{n}$

3. <u>Series-Parallel Configuration</u>



Prob. associated with each series - S_1, S_2, S_3

$n = 2$

R_1 = prob. that s/m is functioning correctly

$= (1-P_2)^n - P_3^n$

$= (1-S_2)^2 - S_3^2$

$= (1-S_2-S_3)(1-S_2+S_3)$

$= (S_1)(1-S_2+S_3)$

$= (S_1)(S_1+S_3+S_3)$

$= S_1^2 + 2S_3 \cdot S_1$

Q. Communication channel's pulse rate = 12 pulses/µsec.

$P(E) = 0.001$ $\qquad P(\bar{E}) = 0.999$

Compute the prob of

a) No errors per µsecond

$$P = \binom{12}{0} (\cancel{0.001})^{0} (0.999)^{12}$$

b) 1 error per microsecond

$$P = \binom{12}{1} (0.001)^1 (0.999)^{11}$$

c) atleast one error per microsecond

$$P = 1 - P(\text{zero error})$$

$$= 1 - \binom{12}{0} (0.001)^0 (0.999)^{12}$$

d) exactly 2 errors per microsecond

$$P = \binom{12}{2} (0.001)^2 (0.999)^{10}$$

7. $R_{m/n}$

$n = 3 \qquad m = 1, 2, 3$

i) $m = 1$ (parallel redundancy)

$$R_{1/3} = \sum_{k=1}^{3} \binom{3}{k} R^k (1-R)^{3-k}$$

$$= 1 - (1-R)^3$$

ii) $m = 2$ (TMR)

$$R_{2/3} = \sum_{k=2}^{3} \binom{3}{k} R^k (1-R)^{3-k} = 3R^2 - 2R^3$$

iii) $m = 3$ (Series system)

$$R_{3/3} = R^3$$

Review Problems

3. In manufacturing a certain component, 2 types of defects - are D_1 and D_2 are likely to occur.

$P(D_1) = 0.05$ $\qquad$ $P(D_2) = 0.1$

a) Is defective?

$$P(\text{defective}) = P(D_1 \cup D_2)$$
$$= P(D_1) + P(D_2) - P(D_1 \cap D_2)$$
$$= P(D_1) + P(D_2) - P(D_1) \cdot P(D_2)$$
$$= 0.05 + 0.1 - (0.05)(0.1)$$
$$= 0.145$$

b) Does not have either kinds of defects?

$$P(\text{does not have any kind of defect}) = 1 - P(\text{defective})$$
$$= 1 - 0.145$$
$$= 0.855$$

c) $P(\text{has only one kind of defect}) = P(D_1 \cap \bar{D}_2) + P(D_2 \cap \bar{D}_1)$
$$= P(D_1) \cdot P(\bar{D}_2) + P(D_2) \cdot P(\bar{D}_1)$$

$$P(\text{has only one kind of defect} \mid \text{defective}) = \frac{P(D_1) \cdot P(\bar{D}_2) + P(D_2) \cdot P(\bar{D}_1)}{P(\text{defective})}$$
$$= \frac{0.05 \times 0.9 + 0.1 \times 0.95}{0.145}$$
$$= 0.9655$$

4.

Value in μf	No. in box 1	2	3
1.0	10	90	25
0.1	50	30	80
0.01	70	90	120

a) $P(0.1/3) = \frac{80}{25 + 80 + 120} = \frac{80}{225}$

$= 0.3555$

$$P\left(1/0.1\right) = \frac{P(0.1/1)\cdot P(1)}{P(0.1/1)\cdot P(1) + P(0.1/2)\cdot P(2) + P(0.1/3)\cdot P(3)}$$

and, $P(1) = P(2) = P(3) = 1/3$

~~$P(0.1/1) \cdot P(1) =$~~

$P(0.1/1) = \frac{50}{130}$ $\quad P(0.1/2) = \frac{30}{210}$ $\quad P(0.1/3) = \frac{80}{225}$

$$P(1/0.1) = \frac{(50/130) \times (1/3)}{(50/130)(1/3) + (30/210)(1/3) + (80/225)(1/3)}$$

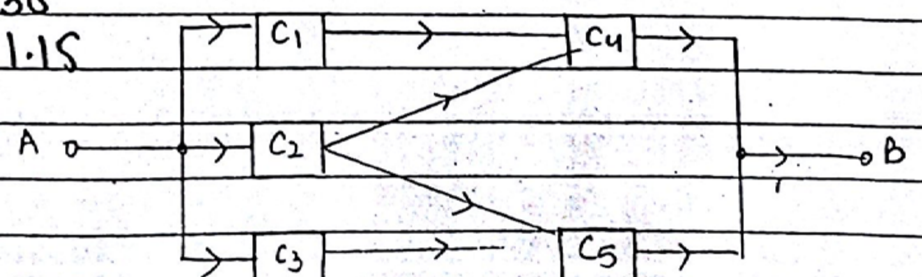
$$= 0.4355$$

ii) $P(1.0/1) = \frac{10}{10+50+70} = \frac{10}{130}$ $\quad P(1.0/2) = \frac{90}{210}$ $\quad P(1.0/3) = \frac{25}{225}$

$P(0.1/1) = \frac{50}{130}$ $\quad P(0.1/2) = \frac{50}{210}$ $\quad P(0.1/3) = \frac{80}{225}$

$P(0.01/1) = \frac{70}{130}$ $\quad P(0.01/2) = \frac{90}{210}$ $\quad P(0.01/3) = \frac{60}{225}$

Pg-38
eg 1.15



Let X_i = i^{th} component is functioning properly

R_i = Reliability of i^{th} component

$R_i = P(X_i)$

~~X = sys is functioning properly~~

~~R = Reliability of the sys~~

C_2 is the component which is connected to every other component in the sys.

Sol: when C_2 is functioning properly

~~$P(X \cap X_2)$~~

~~$P(X|X_2)$ $R(X|X_2)$ $P(X_2)$~~

$P(X) = P(X \cap X_2) + P(X \cap \overline{X_2})$

$\therefore P(X) = P(X/X_2) \cdot P(X_2) + P(X/\overline{X_2}) \cdot P(\overline{X_2})$

$P(X_2) = R_2$ $\qquad P(\overline{X_2}) = 1 - R_2$

$P(X|X_2) = 1 - (1-R_4)(1-R_5)$ (when C_2 is working properly)

$P(X|\overline{X_2}) = 1 - (1-R_1 R_4)(1-R_3 R_5)$

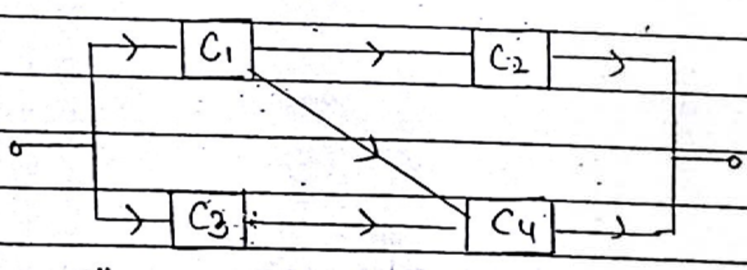
$R_1 R_4$

$R_3 R_5$

So, $P(X) = [1-(1-R_4)(1-R_5)][R_2] + [1-(1-R_1R_4)(1-R_3R_5)][1-R_2]$

$P(X) = 1 - R_2(1-R_4)(1-R_5) - (1-R_2)(1-R_1R_4)(1-R_3R_5)$

Q-40 (2)



Let X_i = i^{th} component is functioning properly

R_i = Reliability of i^{th} component

$R_i = P(X_i)$

X = s/m is functioning properly

R = Reliability of the s/m.

C_1 is the component which is connected to every other component in the s/m.

$P(X) = P(X \cap X_1) + P(X \cap \overline{X_1})$

$= P(X|X_1) \cdot P(X_1) + P(X/\overline{X_1}) \cdot P(\overline{X_1})$

$P(X_1) = R_1$ $\qquad P(\overline{X_1}) = 1 - R_1$

$P(X/X_1) = 1 - (1-R_2)(1-R_4)$

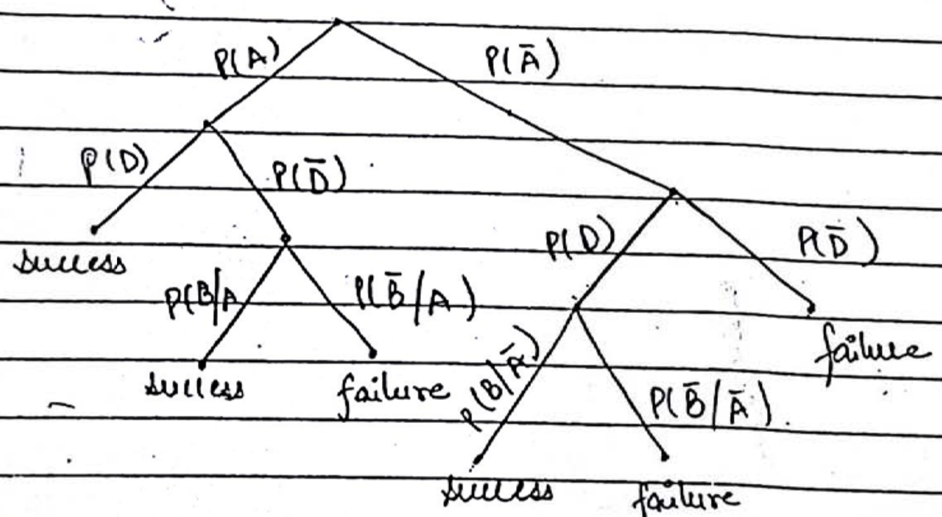
$P(X/\overline{X_1}) = R_3 R_4$

$R(x) = [1-(1-R_3)(1-R_4)]\, R_1 + R_3 R_4 (1-R_1)$

Q39. (1) A = "primary module functions correctly"
B = "alternate module functions correctly"
D = "Detection test following the execution of the primary performs its task correctly."

Assume A & D as well as B & D are independent but A and B are dependent.

Solⁿ



$P(\text{failure}) = P(A)\cdot P(\bar{D})\cdot P(\bar{B}/A) + P(\bar{A})\cdot P(D)\, P(\bar{B}/\bar{A}) + P(\bar{A})\cdot P(\bar{D})$

Pg-23

(5) There are n distinct jobs and n distinct processors. Find the probability that exactly one processor will be idle.

$|S| = n^n$

Solution: Let us number the processors from $1, 2, \cdots n$

Let processor no. 1 be idle

~~So, there will be (n-1) processors available and 'n' jobs will be available~~

Let processor no 2 has been assigned 2 jobs
$(n-2)$ processors will be available and
$(n-2)$ jobs will be available.

In general
Event that any processor can be kept idle
$= \binom{n}{1}$

Event that any processor can be assigned 2 jobs
$= \binom{n-1}{1}$

Event that which 2 jobs are assigned to the processor
$= \binom{n}{2}$

There will be $(n-2)!$ ways to distribute $(n-2)$ jobs to $(n-2)$ processors.

So,
P(exactly 1 processor is kept idle) $= \dfrac{\binom{n}{1}\binom{n-1}{1}\binom{n}{2}(n-2)!}{n^n}$