10/01/11

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BT-3/DX **8347**

MATHEMATICS–III

Paper : Math–201(E)

Time : Three Hours] [Maximum Marks : 160

Note : Attempt *five* questions in all, selecting at least *one* question from each unit. All questions carry equal marks.

UNIT–I

1. (a) Obtain a Fourier series for the function $f(x) = x \sin x,\ 0 < x < 2\pi$.

   (b) Express $f(x) = x$ as a half range cosine series in $0 < x < 2$.

2. (a) Find the Fourier transform of $e^{-x^2/2},\ -\infty < x < \infty$.

   (b) State and prove Convolution theorem for Fourier transforms.

UNIT–II

3. (a) If $\tan(\theta + i\phi) = \tan\alpha + i\sec\alpha$, prove that

$$e^{2\phi} = \pm \cot\frac{\alpha}{2} \text{ and } 2\theta = \left(n + \frac{1}{2}\right)\pi + \alpha.$$

   (b) Define an analytic function state and prove Cauchy-Riemann conditions for an analytic function.

4. (a) If $f(z) = u + iv$ is an analytic function, find $f(z)$, if $u - v = e^x(\cos y - \sin y)$.

   (b) If $\omega = \phi + i\psi$ represents the complex potential for an electric field and $\psi = x^2 - y^2 + \frac{x}{x^2 + y^2}$, determine the function ϕ.

UNIT–III

5. (a) In a bolt factory, machines A, B and C manufacture 25%, 35% and 40% of the total of their output 5%, 4% and 2% are defective bolts. A bolt is drawn at random from the product and is found to be defective. What are the probabilities that it was manufacture by machines A, B or C ?

(b) Find the moment generating function of the exponential distribution $f(x)=\frac{1}{c}\,e^{-x/c}$, $0 \le x \le \infty$, $c>0$. Hence find its mean and S.D.

6. (a) In sampling a large number of parts manufactured by a machine, the mean number of defectives in a sample of 20 is 2. Out of 1000 such samples, how many would be expected to contain at least three defective parts.

(b) In a normal distribution, 31% of the items are under 45 and 8% are over 64. Find the Mean and S.D. of the distribution.

UNIT–IV

7. (a) Solve the following LPP Graphically :

Maximize $Z = 5x_1 + 3x_2$

subject to $4x_1 + 5x_2 \le 1000$,

$3x_1 + 8x_2 \le 1200$,

$5x_1 + 2x_2 \le 1000$;

$x_1, x_2 \ge 0$.

(b) A firm manufactures two items A and B. It purchases castings which are then machined, bored and polished. Castings for items A and B cost Rs. 3 and Rs. 4 each

and are sold at Rs. 6 and Rs. 7 respectively. Running costs of these machines are Rs. 20, Rs. 14 and Rs. 17.50 per hour respectively. Formulate the problem so that the product mix maximizes the profit. The capacities of the machines are

	Item–A	Item–B
Machining	25 per hr.	40 per hr.
Boring	28 per hr.	35 per hr.
Polishing	35 per hr.	25 per hr.

8. (a) Using Simplex method

Maximize $Z = 5x_1 + 3x_2$

subject to $x_1 + x_2 \leq 2,$

$5x_1 + 2x_2 \leq 10,$

$3x_1 + 8x_2 \leq 12;$

$x_1, x_2 \geq 0.$

(b) Using dual Simplex method

Maximize $Z = -3x_1 - x_2$

subject to $x_1 + x_2 \geq 1,$

$2x_1 + 3x_2 \geq 2,$

$x_1, x_2 \geq 0.$
