

BT-4/M11

8459

MATHEMATICS—III

Paper—Math-201E

Time : Three Hours]

[Maximum Marks : 100

Note :—Attempt any **FIVE** questions, selecting at least **ONE** question from each unit. All questions carry equal marks.

UNIT—I

1. (a) Obtain the Fourier series for

$$f(x) = \pi x, 0 \leq x \leq 1$$

$$= \pi(2 - x), 1 \leq x \leq 2$$

and hence deduce that

$$\frac{1}{1^2} + \frac{1}{3^2} + \frac{1}{5^2} + \dots = \pi^2/8.$$

- (b) Show that the Fourier series for
- $\sin ax, -\pi \leq x \leq \pi$
- is

$$\frac{2 \sin a\pi}{\pi} \left\{ \frac{\sin x}{1^2 - a^2} - \frac{2 \sin 2x}{2^2 - a^2} + \frac{3 \sin 3x}{3^2 - a^2} \dots \right\}.$$

2. Solve :

$$\frac{\partial u}{\partial t} = 2 \frac{\partial^2 u}{\partial x^2}$$

if $u(0, t) = 0, u(x, 0) = e^{-x} (x > 0), u(x, t)$ is bounded where $x > 0, t > 0$.

UNIT—II

3. (a) If
- $(a + ib)^p = m^{x+iy}$
- , prove that one of the value of
- y/x
- is
- $2 \tan^{-1}(b/a) + \log(a^2 + b^2)$
- .

- (b) An electrostatic field in the xy -plane is given by the potential function $\phi = 3x^2y - y^3$, find the stream function.
4. (a) If $f(z)$ is an analytic function of z , prove that
- $$\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} \right) \log |f(z)| = 0.$$
- (b) Show that under the transformation $w = \frac{1}{z}$, the circle $x^2 + y^2 - 6x = 0$ is transformed into a straight line in w -plane.

UNIT—III

5. (a) Two persons A and B fire at a target independently and have a probability 0.6 and 0.7 respectively of hitting the target. Find the probability that the target is destroyed.
- (b) Three machines M_1 , M_2 and M_3 produce indential items. Of their respective outputs 5%, 4% and 3% items are faulty. On a certain day M_1 has produced 25% of the total output, M_2 has produced 30% and M_3 the remainder. An item selected at random is found to be faulty. What are the chances that it was produced by the machine with heighest output ?
6. (a) Fit the binomial distribution to the following data :—

x	f
0	2
1	14
2	20
3	34
4	22
5	8

- (b) Show that the standard deviation for a normal distribution is approximately by 25% more than the mean deviation.

UNIT—IV

7. (a) Using graphical method, solve :

$$\text{Min. : } Z = 20x_1 + 10x_2$$

subject to :

$$x_1 + 2x_2 \leq 40$$

$$3x_1 + x_2 \geq 30$$

$$4x_1 + 3x_2 \geq 60$$

$$x_1, x_2 \geq 0.$$

- (b) Find all the basic solutions to :

$$\text{Max. : } Z = x_1 + 3x_2 + 3x_3$$

subject to :

$$x_1 + 2x_2 + 3x_3 = 4,$$

$$2x_1 + 3x_2 + 5x_3 = 7 \text{ and}$$

$$x_1, x_2, x_3 \geq 0$$

Which of the basic solutions are :

- (i) Non-degenerate basic feasible
- (ii) Optimal basic feasible ?

8. (a) Use Simplex method, to solve :

$$\text{Max. : } Z = x_1 + x_2 + 3x_3$$

subject to :

$$3x_1 + 2x_2 + x_3 \leq 3$$

$$2x_1 + x_2 + 2x_3 \leq 2$$

$$x_1, x_2, x_3 \geq 0.$$

(b) Using dual simplex method, solve :

$$\text{Min. : } Z = 2x_1 + x_2$$

subject to :

$$3x_1 + x_2 \geq 3$$

$$4x_1 + 3x_2 \geq 6$$

$$x_1 + 2x_2 \leq 3$$

$$x_1, x_2 \geq 0.$$