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Total No. of Pages : 3

BT-3/D05

8424

MATHEMATICS-III

Paper-MATH—201E

Time : Three Hours]

[Maximum Marks : 100

Note :— Attempt any FIVE questions in all selecting at least One question from each unit. All questions carry equal marks.

UNIT—I

- (a) If $f(x) = 0$ for $-\pi < x < 0$,
 $= \sin x$ for $0 < x < \pi$.

Prove that $f(x) = \frac{1}{\pi} + \frac{\sin x}{2} - \frac{2}{\pi} \sum_{m=1}^{\infty} \frac{\cos 2mx}{4m^2 - 1}$.

Hence show that $\frac{1}{1.3} - \frac{1}{3.5} + \frac{1}{5.7} - \dots \infty = \frac{1}{4}(\pi - 2)$.

- (b) If $f(x) = \sin x$ for $0 \leq x < \pi$,
 $= \cos x$ for $\pi/4 \leq x \leq \pi/2$

Expand $f(x)$ in a series

2. (a) Solve the integral equation $\int_0^{\infty} f(x) \cos \alpha x dx = e^{-x}$.

- (b) Verify convolution theorem for $f(x) = g(x) = e^{-x}$.

UNIT—II

3. (a) If $\cosh x = \sec \theta$, prove that $\tanh^2 \frac{x}{2} = \tan^2 \frac{\theta}{2}$.

- (b) Reduce $\tan^{-1}(\cos \theta + i \sin \theta)$ to the form $a + ib$. Hence show that

$$\tan^{-1}(e^{i\theta}) = \frac{i\pi}{2} + \frac{\pi}{4} - \frac{i}{2} \log \tan \left(\frac{\pi - \theta}{4} \right)$$

Contd.

4. (a) Find the regular function whose imaginary part is

(i) $e^x \sin y$

(ii) $\frac{2 \sin x \sin y}{\cos 2x + \cosh 2y}$

(b) Discuss fully the transformation $w = c \cosh z$, where c is a real number. What physical problem can we study with the help of this transformation?

UNIT—III

5. (a) In a bolt factory, machines A, B and C manufacture 25%, 35% and 40% of the total of their output 5%, 4% and 2% are defective bolts. A bolt is drawn at random from the product and is found to be defective. What are the probabilities that it was manufactured by machines A, B or C?

(b) A function is defined as under :

$$f(x) = \frac{1}{k}, \quad x_1 \leq x \leq x_2$$
$$= 0 \quad \text{elsewhere.}$$

Find the cumulative distribution of the variable x when k satisfies the requirements for $f(x)$ to be a density function.

6. (a) In a lot of 500 solenoids 25 are defective, find the probability of 0, 1, 2, 3 defective solenoids in a random sample of 20 solenoids.

(b) Fit a Poisson distribution to the set of observations :

$x :$	0	1	2	3	4
$f :$	122	60	15	2	1

UNIT—IV

7. (a) A firm manufactures 3 products A, B and C. The profits are Rs. 3, Rs. 2 and Rs. 4 respectively. The firm has two

machines M_1 and M_2 and below is the required processing time in minutes for each machine on each product :

	PRODUCT		
	A	B	C
Machine M_1	4	3	5
Machine M_2	2	2	4

Machine M_1 and M_2 have 2000 and 2500 machine minutes respectively. The firm must manufacture 100 A's 200 B's and 50 C's but not more than 150 A's. Set up an L.P.P. to maximize the profit.

(b) Convert the LPP below to a standard form :

$$\text{Maximize } Z = 3x_1 - 2x_2 + 4x_3$$

$$\text{subject to } x_1 + 2x_2 + x_3 \leq 8,$$

$$2x_1 - x_2 + x_3 \geq 2,$$

$$4x_1 - 2x_2 - 3x_3 = -6 ; x_1, x_2 \geq 0.$$

8. (a) Using Simplex method

$$\text{Maximize } z = 5x_1 + 3x_2$$

$$\text{subject to } x_1 + x_2 \leq 2, 5x_1 + 2x_2 \leq 10,$$

$$3x_1 + 8x_2 \leq 12 ; x_1, x_2 \geq 0.$$

(b) Using dual Simplex method,

$$\text{Maximize } Z = -3x_1 - x_2$$

$$\text{subject to } x_1 + x_2 \geq 1,$$

$$2x_1 + 3x_2 \geq 2, \quad x_1, x_2 \geq 0.$$