204 (B)

Roll No. ..... Total Pages : 3

**8103**

**BT-3/D09**

DISCRETE STRUCTURES

Paper : CSE-205(E)

Time : Three Hours] [Maximum Marks : 100

**Note :** Attempt *five* questions in all, selecting at least *one* question from each unit.

## UNIT–I

**1.** (a) Let A, B, C be arbitrary sets, prove that :

(i) $(A - B) - C = (A - C) - B$.

(ii) $(A - B) - C = (A - C) - (B - C)$.

(iii) $(A \cap B) \subseteq B \subseteq (A \cup B)$.

(iv) $(A \cap B) \subseteq A \subseteq (A \cup B)$.

(b) Show that if any four numbers are chosen from 1 to 6, then two of them will add to 7. (12,8)

**2.** (a) Define the following :

(i) Reflexive relation.

(ii) Anti-symmetric relation.

(iii) Transitive relation.

(iv) Irreflexive relation.

(b) Let P(S) be the power set of the set $S = \{1, 2, 3\}$. Deduce the relation set and construct the hasse diagram using lattice $(P(S), \cup, \cap)$.

(c) Find the greatest and smallest elements of the lattice of Question No. 2(b). (8,8,4)

## UNIT–II

3. (a) Construct the truth tables for the following statements :

(i) $p \rightarrow \mathrm{P}$.

(ii) $(p \rightarrow (q \rightarrow r)) \rightarrow ((p \rightarrow q) \rightarrow (p \rightarrow r))$.

(b) Determine the number of ways to place $(2t + 1)$ distinguishable balls in three distinct boxes so that any two boxes together will contain more balls than the other. (8,12)

4. (a) Solve the difference equation :

$a_r - {}^{7}a_{r-1} + {}^{10}a_{r-2} = 0$ satisfying the conditions, $a_0 = 0$ and $a_1 = 6$.

(b) John made the following statements :

(i) I love Lucky.

(ii) If I love Lucky, then I also love Vivian.

Given that John either told the truth or lied in both cases, determine whether John really loves Lucky. (10,10)

## UNIT–III

5. (a) For any arbitrary group G, prove :

(i) identity element is unique.

(ii) inverse of an element is unique.

(iii) $\left(a^{-1}\right)^{-1} = a$.

(iv) $(ab)^{-1} = b^{-1}a^{-1}$.

(b) Let G be a finite group and H be a subgroup of G. If $a\mathrm{H} = \{ah : h \in \mathrm{H}\}$, show that for any $a, b \in \mathrm{G}$, either $a\mathrm{H} = b\mathrm{H}$ or $a\mathrm{H} \cap b\mathrm{H} = \phi$. (10,10)

6. (a) Is addition (+) on the set $s = \{0, 1, -1\}$ an operation. Why ?

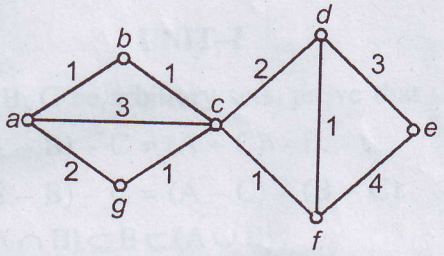
(b) State and prove Lagrange's Theorem.

(c) Prove that $Z_p$ is a field, where $p$ is a prime number.

(4,8,8)

## UNIT–IV

7. (a) Find the shortest path from $a$ to $e$ in the following weighted graph using Dijkstra's algorithm :




(b) Prove that there is one and only one path from root to any node in a tree. (14,6)

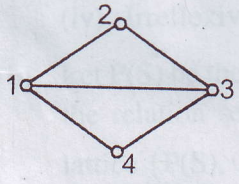
8. (a) Construct the binary tree for the expression

$$E = (a+b) * ((d-e)/(f-g)) - h/k$$

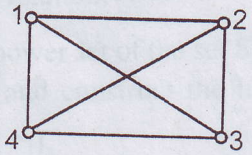
and find its preorder and postorder traversals.

(b) Prove that $K_{3,3}$ is non-planar.

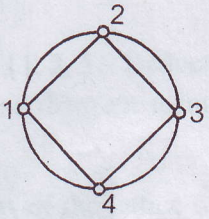
(c) Check and state which of the following are bi-partite graphs and why ?




(i)




(ii)




(iii) (8,6,6)