Roll No. .... **Total No. of Pages : 3**

BT-4/JX 8394

Mathematics—III

Paper : Math-201-E

Time : Three Hours] [Maximum Marks : 100

Note :— Attempt any **FIVE** questions, selecting **ONE** question from each section.

SECTION—I

1. (a) Draw the graph of the function

$$f(x) = 0, \quad -\pi < x < 0$$
$$= x^2, \quad 0 < x < \pi.$$

If $f(2\pi + x) = f(x)$, obtain the Fourier series of $f(x)$. 10

(b) Find the half range cosine series for

$$f(x) = kx, \quad 0 \le x \le l/2$$
$$= k(l - x), \quad l/2 \le x \le l.$$

Hence find the sum of the series

$$\frac{1}{1^2} + \frac{1}{3^2} + \frac{1}{5^2} + \ldots\ldots \text{ to } \infty.$$ 10

2. (a) Find the Fourier cosine transform of $f(x) = \dfrac{1}{1+x^2}$. 10

(b) Using suitable Fourier transform solve the equation $\dfrac{\partial u}{\partial t} = \dfrac{\partial^2 u}{\partial x^2}$, $(x > 0, t > 0)$ subject to conditions :

(i) $u = 0$ when $x = 0, t > 0$.

(ii) $u = \begin{cases} 1, & 0 < x < 1 \\ 0, & x \ge 1. \end{cases}$

(iii) $u(x, t)$ is bounded. 10

SECTION—II

3. (a) If $\tan(x + iy) = \sin(u + iv)$, prove that

$$\frac{\sin 2x}{\sinh 2y} = \frac{\tan u}{\tanh v}.$$ 10

(b) If f(z) is an analytic function with constant modulus, show that f(z) is constant. 10

4. (a) Determine the analytic function whose real part is

$$u = x^3 - 3xy^2 + 3x^2 - 3y^2 + 1.$$ 10

(b) Show that the bilinear transformation $w = (2z + 3) / (z - 4)$ maps the circle $x^2 + y^2 - 4x = 0$ into the line $4u + 3 = 0$. 10

SECTION—III

5. (a) A and B throw alternately with a pair of dice. The one who throws 9 first wins. Show that the chances of their winning are 9 : 8. 10

(b) The contents of three urns are : 1 white, 2 red, 3 green balls; 2 white, 1 red, 1 green balls and 4 white, 5 red, 3 green balls. Two balls are drawn from an urn chosen at random. These are found to be one white and one green. Find the probability that the balls so drawn came from the third urn. 10

6. (a) If a random variable has a Poisson distribution such that $P(1) = P(2)$, find

(i) mean of the distribution

(ii) $P(4)$. 10

(b) Show that the S.D. for a normal prob. distribution is approximately 25% more than the mean deviation. 10

SECTION—IV

7. (a) Using graphical method, solve :

Maximize $Z = 100x + 40y$

subject to $10x + 4y \leq 2000$,

$3x + 2y \leq 900$,

$6x + 12y \leq 3000$ and

$x, y \geq 0$. 10

(b) Convert the given L.P.P. to standard form :

Max. $Z = 3x_1 - 2x_2 + 4x_3$

subject to $x_1 + 2x_2 + x_3 \le 8,$

$2x_1 - x_2 + x_3 \ge 2,$

$4x_1 - 2x_2 - 3x_3 = -6,$

$x_1, x_2 \ge 0.$ 10

8. (a) Using simplex method, solve :

Maximize $P = 10x + y + 2z$

subject to $x + y - 2z \le 10,$

$4x + y + z \le 20,$

$x, y, z \ge 0.$ 10

(b) Using dual simplex method, solve :

Min. $Z = 2x_1 + x_2$

subject to $3x_1 + x_2 \ge 3,$

$4x_1 + 3x_2 \ge 6,$

$x_1 + 2x_2 \le 3,$

$x_1, x_2 \ge 0.$ 10