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BT-4/M-12

MATHEMATICS–III

Paper–MATH-201-E

Time Allowed : 3 Hours] [Maximum Marks : 100

Note : Attempt **five** questions in all, selecting at least **one** question from each Unit. All questions carry equal marks.

UNIT–I

1. (a) Obtain a Fourier series to represent $f(x) = \left(\frac{\pi - x}{2}\right)^2$ in the range 0 to 2π.

   (b) If $f(x) = \begin{cases} x, & 0 < x < \pi/2 \\ \pi - x, & \pi/2 < x < \pi, \end{cases}$

   expand f(x) in a half range cosine series.

2. (a) Find the Fourier transform of

$$f(x) = e^{-x^2/2}, \ -\infty < x < \infty.$$

(b) Obtain Fourier sine transform of

$$f(x)=\begin{cases} x, & 0<x<1 \\ 2-x, & 1<x<2 \\ 0, & x>2 \end{cases}$$

UNIT–II

3. (a) Separate $\cos^{-1}(\cos\theta + i\sin\theta)$ into real and imaginary parts, where θ is a positive acute angle.

(b) Determine the analytic function, whose real part is $4 = y + e^x \cos y$.

4. (a) If $f(z)$ is an analytic function with constant modulus, show that $f(z)$ is constant.

(b) Determine the region of the w-plane into which the region bounded by $x = 1, y = 1, x + y = 1$ is mapped by the transformation $w = z^2$.

UNIT–III

5. (a) A speaks the truth in 75% cases and B in 80% of the cases. In what percentage of cases, are they likely to contradict each other in stating the same fact ?

(b) The contents of three urns are 1 white, 2 red, 3 green balls; 2 white, 1 red, 1 green balls and 4 white, 5 red, 3 green balls. Two balls are drawn from an urn chosen at random. These are found to be one white and one green. Find the probability that the balls so drawn came from the third urn.

6. (a) A source of liquid is known to contain bacteria with the mean number of bacteria per cubic centimetre equal to 3. Ten I.C.C. test tubes are filled with the liquid. Assuming that Poisson distribution is aplicable, calculate the probability that all the test tubes will show growth i.e. contain at least one bacterium each.

(b) The mean and standard deviation of the marks obtained by 1,000 students in an examination are 34.4 and 16.5 respectively. Assuming the normality of the distribution, find the approximate number of students expected to obtain marks between 30 and 60.

UNIT–IV

7. (a) Using graphical method, solve the following L.P. problem :

Maximize $Z = 100x_1 + 40x_2$

Subject to
$$10x_1 + 4x_2 \le 2000$$
$$3x_1 + 2x_2 \le 900$$
$$6x_1 + 12x_2 \le 3000$$
$$x_1, x_2 \ge 0.$$

(b) Convert the following L.P.P. to standard form :

Maximize $Z = 3x_1 - 2x_2 + 4x_3$

Subject to
$$x_1 + 2x_2 + x_3 \le 8$$
$$2x_1 - x_2 + x_3 \ge 2$$
$$4x_1 - 2x_2 - 3x_3 = -6$$
$$x_1, x_2 \ge 0.$$

8. (a) Use simplex method, to solve :

$$\text{Maximize } Z = 10x_1 + x_2 + 2x_3$$

Subject to

$$x_1 + x_2 - 2x_3 \leq 10$$

$$4x_1 + x_2 + x_3 \leq 20$$

$$x_1, x_2, x_3 \geq 0.$$

(b) Use dual simplex method, to solve :

$$\text{Minimize } Z = x_1 + 2x_2 + 3x_3$$

Subject to

$$2x_1 - x_2 + x_3 \geq 4$$

$$x_1 + x_2 + 2x_3 \leq 8$$

$$x_2 - x_3 \geq 2$$

$$x_1, x_2, x_3 \geq 0.$$