BT-4/M-13

MATHEMATICS–III

Paper : MATH-201(E)

Time : Three Hours] [Maximum Marks : 100

Note : Attempt *five* questions in all, selecting at least *one* question from each unit.

UNIT–I

1. (a) Obtain Fourier series of the function $f(x) = x - x^2$ from $x = -\pi$ to $x = \pi$.

   (b) Express $f(x) = x$ as a half range cosine series in $0 < x < 2$.

2. (a) Solve the integral equation

$$\int_0^\infty f(\theta) \cos \alpha\theta \, d\theta = \begin{cases} 1-\alpha, & 0 \le \alpha \le 1 \\ 0, & \alpha > 1 \end{cases}.$$

   Hence, evaluate $\int_0^\infty \frac{\sin^2 t}{t^2} dt$.

   (b) Using Parseval's identity, prove that

$$\int_0^\infty \frac{dt}{(a^2+t^2)(b^2+t^2)} = \frac{\pi}{2ab(a+b)}.$$

UNIT–II

3. (a) Separate $\tan^{-1}(x+iy)$ into real and imaginary parts.

   (b) Prove that $\tan\left[i \log\left(\frac{a-ib}{a+ib}\right)\right] = \frac{2ab}{a^2-b^2}$.

4. (a) Show that the function defined by $f(z) = \sqrt{|xy|}$ is not analytic at the origin even though C.R. equations are satisfied thereof.

(b) Find the bilinear transformation which maps the points $z = 1, i, -1$ on to the points $w = i, 0, -i$.

UNIT–III

5. (a) Given that $P(A) = 1/4$, $P(AB) = 1/3$ and $P(A \cup B) = 1/2$, evaluate $P(A/B)$, $P(B/A)$, $P\{A \cap B^C)$, $P(A/B^C)$.

(b) The frequency function of a continuous random variable is given by $f(x) = y_0 x(2 - x)$, $0 \le x \le 2$. Find the value of y_0, mean and variance of x.

6. (a) The probability that a bomb dropped from a plane will strike the target is 1/5. If six bombs are dropped, find the probability that (i) exactly two will strike the target, (ii) at least two will strike the target.

(b) Fit a Poisson distribution to the following data :

x	:	0	1	2	3	4
f	:	46	38	22	9	1

UNIT–IV

7. (a) Use Graphical method to solve the LPP :

Max. $Z = 3x_1 + 4x_2$,

subject to $4x_1 + 2x_2 \le 80$,

$2x_1 + 5x_2 \le 180$,

$x_1, x_2 \ge 0$.

(b) Use Dual Simplex method to solve the LPP :

Min. $Z = x_1 + 2x_2 + 3x_3$,

subject to $2x_1 - x_2 + x_3 \geq 4$,

$x_1 + x_2 + 2x_3 \leq 8$,

$x_2 - x_3 \geq 2$,

$x_1, x_2, x_3 \geq 0$.

8. (a) Find all basic, feasible, degenerate solutions to the following problem :

Max. $Z = x_1 + 3x_2 + 3x_3$,

subject to $x_1 + 2x_2 + 3x_3 = 4$,

$2x_1 + 3x_2 + 5x_3 = 7$,

$x_1, x_2, x_3 \geq 0$.

(b) Use Simplex method to solve the LPP :

Max. $Z = 5x_1 + 3x_2$

subject to $x_1 + x_2 \leq 2$,

$5x_1 + 2x_2 \leq 10$,

$3x_1 + 8x_2 \leq 12$,

$x_1, x_2, \geq 0$.