8. (a) Construct the following graphs : 8

(i) Eulerian but not Hamiltonian.

(ii) Hamiltonian but not Eulerian.

(iii) Neither Eulerian nor Hamiltonian.

(iv) Eulerian and Hamiltonian.

(b) Define : Graph, Simple Graph, Pseudo graph and Weighted graph. 8

(c) A tree of order n has size $(n-1)$. Prove. 4

Roll No. .... Total Pages : 4

8303

BT-3/DX

DISCRETE STRUCTURE

Paper : CSE–205(E)

Time : Three Hours] [Maximum Marks : 100

Note : Attempt *five* questions in all, selecting at least *one* question from each section.

SECTION–I

1. (a) If A, B and C be subsets of the universal set U, then prove : 14

(i) $A \cap (B \,\Delta\, C) = (A \cap B) \,\Delta\, (A \cap C)$.

(ii) $A \times (B \cap C) = (A \times B) \cap (A \times C)$.

(b) In a city 60% of the residents can speak German and 50% can speak French. What percentage of residents can speak both the languages, if 20% residents can not speak any of these language ? 6

2. (a) (i) If R be an equivalence relation defined on a non-empty set A and x, y be arbitrary elements in A, and $x \in [x]$ and $y \in [x]$, then $[x] = [y]$. 5

(ii) Prove by method of induction 5

$$1^3 + 2^3 + 3^3 + \dots\dots\dots + n^3 = \left[\frac{n(n+1)}{2}\right]^2.$$

(b) (i) Let $f : A \to B$ and $g : B \to C$ be two functions, then (gof) is one-one if both f and g are one-one and (gof) is onto if both f and g are onto. 5

(ii) Give an example of a function which is (α) Injective but not Surjective (β) Bijective (γ) Surjective but not Injective, (δ) Constant. 5

SECTION–II

3. (a) Prove by constructing truth table 6

$$P \to (Q \vee R) \equiv (P \to Q) \vee (P \to R).$$

(b) Solve the recurrence relation $S_n - 7S_{n-1} + 10S_{n-2} = 0$, $S_0 = 0$ and $S_1 = 3$ by using generating function where, $n \geq 2$. 7

(c) Find the total solution of the difference equation

$S_n - S_{n-1} = 5$, given that $S_0 = 2$. 7

4. (a) Solve the difference equation

$$\sqrt{S_{n-1}+\sqrt{S_{n-2}+\sqrt{S_{n-3}+\sqrt{\ldots\ldots\ldots}}}}, \text{ given that } S_0 = 4.$$ 7

(b) Find the total distinct numbers of six digits that can be formed with 0, 1, 3, 5, 7 and, 9 and how many of them is divisible by 10 ? 6

(c) Discuss the importance of recurrence relations in the binary algorithm. 7

SECTION–III

5. (a) If G is a set of Real numbers (non-zero) and let

$a * b = \dfrac{a.b}{2}$, show that (G, *) is an abelian group. 7

(b) A finite integral domain is a field. Prove. 6

(c) State and prove Lagrange's theorem. 7

6. (a) Let R is a ring with unity and $(x \cdot y)^2 = x^2 \cdot y^2 \ \forall \ x, y \in R$. Show that R is a commutative ring. 7

(b) Show that the characteristic of an integral domain is either O or a prime number. 6

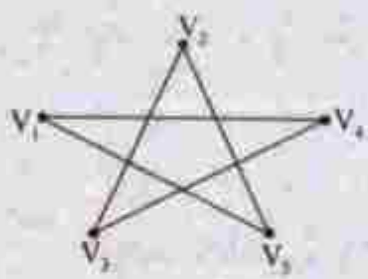
(c) If H is subgroup of a group G and $h \in H$, then $Hh = H = hH$. 7

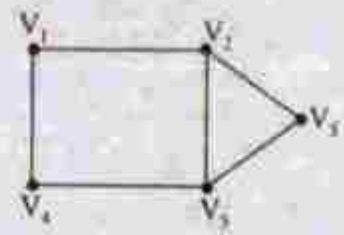
SECTION–IV

7. (a) Determine whether the graph given below by its adjacency matrix is connected or not, where the matrix

$$A = \begin{pmatrix} 0 & 1 & 1 & 1 \\ 1 & 0 & 1 & 0 \\ 1 & 1 & 0 & 1 \\ 1 & 0 & 1 & 0 \end{pmatrix}.$$ 7

(b) Find the complement of the following graphs :





6

(c) If T is a binary tree of height h and order p, then $(h + 1) \leq p \leq 2^{(h+1)} - 1$. 7