

BT-3 / D-13

MATHEMATICS-III

Paper-MATH-201E

Time allowed : 3 hours]

[Maximum marks : 100

Note : Attempt any five questions in all, selecting at least one question from each unit. All questions carry equal marks.

Unit-I

1. (a) Obtain the Fourier series for the function

$$f(x) = x^2, \quad -\pi < x < \pi$$

Hence show that

$$\frac{1}{1^2} + \frac{1}{2^2} + \frac{1}{3^2} + \dots = \sum \frac{1}{n^2} = \frac{\pi^2}{6} \quad 10$$

- (b) Find the half range cosine series for the function $f(x) = x^2$ in the range $0 \leq x \leq \pi$ 10

2. (a) Using Fourier Integral representation, show that

$$\int_0^\infty \frac{w \sin x w}{1 + w^2} dw = \frac{\pi}{2} e^{-x} \quad (x > 0) \quad 10$$

- (b) State and prove convolution theorem. 10

(2)

Unit-II

3. (a) If $\rho = \log \tan \left(\frac{\pi}{4} + \frac{\theta}{2} \right)$, prove that

$$\tanh \left(\frac{u}{2} \right) = \tan \frac{\theta}{2}$$

10

$$\theta = -i \log \tan \left(\frac{\pi}{4} + \frac{i u}{2} \right)$$

- (b) If $\cos(\alpha + i\beta) = r(\cos \theta + i \sin \theta)$, prove that

$$e^{2\beta} = \sin(\alpha - \theta) / \sin(\alpha + \theta)$$

10

4. (a) If $f(z)$ is a regular function of z , prove that

$$\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} \right) |f(z)|^2 = 4 |f'(z)|^2$$

10

- (b) Determine the analytic function whose real part is $e^x \sin y$.

10

Unit-III

5. (a) A random variable x has the following probability function:

x	:	-2	-1	0	1	2	3
$f(x)$:	0.1	k	0.2	$2k$	0.3	k

Find k , mean, variance and standard deviation.

10

(3)

- (b) In a bolt factory, machines A, B and C manufacture 25%, 35% and 40% of the total of their output 5%, 4% and 2% are defective bolts. A bolt is drawn at random and is found to be defective. What is the probability that it was manufactured by A, B or C? 10

6. (a) If the probability of a bad reaction from a certain injection is 0.001, determine the chance that out of 2,000 individuals more than two will get a bad reaction. 10
- (b) In a normal distribution, 31% of the items are under 45% and 8% are over 64%. Find the mean and standard deviation. 10

Unit-IV

- ✓ (a) Using graphical method, solve the L.P.P.

$$\text{Min } Z = 20x_1 + 30x_2$$

subject to the constraints

$$x_1 + 2x_2 \leq 40$$

$$3x_1 + x_2 \geq 30$$

$$4x_1 + 3x_2 \geq 60$$

$$x_1, x_2 \geq 0$$

10

- (b) Obtain all the basic solutions to the following system of linear equations:

$$x_1 + 2x_2 + x_3 = 4$$

$$2x_1 + x_2 + 5x_3 = 5$$

which of these are feasible and non-degenerate? 10

8/ (a) / Using Simplex method, solve the LPP

$$\text{Min } Z = 3x_1 + 5x_2 + 4x_3$$

subject to the constraints

$$2x_1 + 3x_2 \leq 8$$

$$2x_2 + 5x_3 \leq 10$$

$$3x_1 + 2x_2 + 4x_3 \leq 15$$

$$x_1, x_2, x_3 \geq 0$$

10

(b) / Using Dual Simplex method, solve the L.P.P.

$$\text{Min } Z = 2x_1 + 2x_2 + 4x_3$$

sub. to constraints

$$2x_1 + 3x_2 + 5x_3 \geq 2$$

$$3x_1 + x_2 + 7x_3 \leq 3$$

$$x_1 + 4x_2 + 6x_3 \leq 5$$

$$x_1, x_2, x_3 \geq 0$$

10