

BT4/M06

8755

Mathematics-III

Paper : MaE-201/202

Time : Three Hours]

[Maximum Marks : 75

Note :- Attempt FIVE questions, selecting at least ONE question from each part.

PART-A

(a) Evaluate $\Delta^2 \left\{ \frac{5x+12}{x^2+5x+16} \right\}$ interval of differencing being

unity.

(b) Apply Bessel formula to obtain Y_{35} given that $Y_{30} = 2854$, $Y_{34} = 3162$, $Y_{38} = 3544$, $Y_{42} = 3992$.

(a) D stands for differential $\frac{d}{dx}$, prove that

$$D = \frac{1}{h} \left[\Delta - \frac{\Delta^2}{2} + \frac{\Delta^3}{3} - \dots \right]$$

and hence find $Df(0)$ from the following table :-

x	0	1	2	3	4	5
f(x)	4	8	15	7	6	2

(b) Using Simpson's $\frac{1}{3}$ rule, evaluate $\int_1^2 \sqrt{x - \frac{1}{x}} dx$ correct to two decimal places by dividing the interval in the four equal parts.

3. (a) Solve the difference equation

$$Y_{n+2} + 6Y_{n+1} + 9Y_n = \eta 2^n + (-3)^n.$$

- (b) Using Regula Falsi method find one of the roots of the equation, correct to three decimal places

$$x^3 - 4x - 9 = 0$$

4. (a) Using Newton Raphson method, find the real root of $x \log_{10} x - 1.2 = 0$, correct to five decimal places.

- (b) Using Modified Euler's method, solve $\frac{dy}{dx} = x + y^2$ at $x = 0.6$ in steps of 0.2, given that $y = 1$ at $x = 0$.

PART-B

5. (a) Form a partial differential equation, by eliminating the arbitrary function

$$z = e^{my} \phi(x - y)$$

- (b) Using Charpit's method, solve the partial differential equation

$$p(1 + q) = qz.$$

6. (a) Solve the partial differential equation

$$y^2 p - xyq = x(z - 2y).$$

- (b) Solve the partial differential equation

$$(D^2 + DD' - 6D'^2)z = \cos(2x + y).$$

7. (a) Using the method of separation of variables, solve

$$\frac{\partial^2 u}{\partial x^2} \neq \frac{\partial u}{\partial y} + \partial u, \text{ given that } u = 0, \frac{\partial u}{\partial x} = 1 + e^{-3y}$$

when $x = 0$, for all value of y .

- (b) Find the solution of $\frac{\partial v}{\partial t} = K \frac{\partial^2 v}{\partial x^2}$ having given that $V = V_0 \sin nt$ when $x = 0$, for all value of t and $v = 0$ when x is very large.

PART-C

8. Explain Frobenius method to solve a second order differential equation in series and hence solve

$$xy'' + y' + xy = 0$$

9. (a) Show that

$$\int \frac{-5}{2}(x) = \sqrt{\frac{2}{\pi x}} \left\{ \frac{3}{2} \sin x + \frac{3-x^2}{x^2} \cos x \right\}.$$

- (b) Show that

$$\frac{d}{dx} \left[\int_n^2(x) + \int_{n+1}^2(x) \right] = 2 \left\{ \frac{x}{n} \int_n^2(x) - \frac{n+1}{x} \int_{n+1}^2(x) \right\}.$$

- (c) Show that

$$\int_{-1}^1 P_n^2(x) dx = \frac{2}{2n+1}$$