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MATHEMATICS–III

Paper : Math-201(E)

Time : Three Hours]

[Maximum Marks : 100

Note : Attempt *five* questions in all, selecting at least *one* question from each unit. All questions carry equal marks.

UNIT–I

1. (a) Find a Fourier series to represent $x - x^2$ from $x = -\pi$ to $x = \pi$.

   (b) If $f(x) = |\cos x|$, expand $f(x)$ as a Fourier series in the interval $(-\pi, \pi)$.

2. (a) Find the Fourier transform of

$$f(x) = \begin{cases} 1 & \text{for } |x| < 1 \\ 0 & \text{for } |x| > 1. \end{cases}$$

Hence evaluate $\int_0^\infty \frac{\sin x}{x}\, dx$.

(b) State and prove Convolution theorem for Fourier transforms.

UNIT–II

3. (a) If $\tan(\theta + i\phi) = \tan\alpha + i\sec\alpha$, prove that

$$e^{2\phi} = \pm\cot\frac{\alpha}{2} \text{ and } 2\theta = \left(n + \frac{1}{2}\right)\pi + \alpha.$$

(b) If $f(z)$ is analytic function of z, prove that

$$\left\{\frac{\partial}{\partial x}|f(z)|\right\}^2+\left\{\frac{\partial}{\partial y}|f(z)|\right\}^2=|f'(z)|^2.$$

4. (a) Determine the analytic function whose real part is

$$\log\sqrt{(x^2+y^2)}\,.$$

(b) Show that the polar form of Cauchy-Riemann equ are

$$\frac{\partial u}{\partial r}=\frac{1}{r}\frac{\partial v}{\partial \theta},\ \frac{\partial v}{\partial r}=-\frac{1}{r}\frac{\partial u}{\partial \theta}\,.$$

Deduce that

$$\frac{\partial^2 u}{\partial r^2}+\frac{1}{r}\frac{\partial u}{\partial r}+\frac{1}{r^2}\frac{\partial^2 u}{\partial \theta^2}=0\,.$$

UNIT–III

5. (a) A pair of dice is tossed twice. Find the probability o scoring 7 points

(a) once,

(b) at least once, and

(c) twice.

(b) Fit a Binomial distribution to the following frequency distribution :

x:	0	1	2	3	4	5	6
f:	13	25	52	58	32	16	4

6. (a) In a certain factory turning out razor blades, there is a small chance of 0.002 for any blade to be defective. The blades are supplied in packets of 10, use Poisson distribution to calculate the approximate number of packets containing no defective, one defective and two defective blades respectively in a consignment of 10,000 packets.

(b) Define the Normal distribution and discuss the properties.

UNIT–IV

7. (a) A firm manufactures three products A, B and C. The profits are Rs. 3, Rs. 2 and Rs. 4 respectively. The firm has two machines M_1 and M_2 and below is the required processing time in minutes for each machine on each product :

	Product		
	A	B	C
Machine M_1	4	3	5
Machine M_2	2	2	4

Machine M_1 and M_2 have 2000 and 2500 machine minutes respectively. The firm must manufacture 100 A's, 200 B's and 50 C's but not more than 150 A's. Set up an L.P.P. to maximize the profit.

(b) Convert the LPP below to a standard form

Maximize $Z = 3x_1 - 2x_2 + 4x_3$

subject to $x_1 + 2x_2 + x_3 \leq 8$,

$2x_1 - x_2 + x_3 \geq 2$,

$4x_1 - 2x_2 - 3x_3 = -6$;

and $x_1, x_2 \geq 0$.

8. (a) Using Simplex method

Maximize $Z = 5x_1 + 3x_2$

subject to $x_1 + x_2 + \leq 2$,

$5x_1 + 2x_2 \leq 10$,

$3x_1 + 8x_2 \leq 12$;

and $x_1, x_2 \geq 0$.

(b) Using Dual Simplex method

Maximize $Z = -3x_1 - x_2$

subject to $x_1 + x_2 \geq 1$,

$2x_1 + 3x_2 \geq 2$,

and $x_1, x_2 \geq 0$.
