

BT-4 / M-14

MATHEMATICS-III

Paper-MATH-201 E Opt. I

Time allowed : 3 hours] [Maximum marks : 100

Note : Do any five questions, selecting at least one from each unit. All questions carry equal marks.

Unit-I

1. (a) Find the Fourier series of periodicity 2 for

$$f(x) = \begin{cases} x & \text{in } -1 < x \leq 0 \\ x+2 & \text{in } 0 < x < 1 \end{cases}$$

- (b) Obtain the half-range sine series in the interval $0 < x < \pi$ for

$$\text{the function } f(x) = \frac{e^{3x} - e^{-3x}}{e^{3x} + e^{-3x}}.$$

2. (a) Find the Fourier cosine transform of e^{-ax} . Hence evaluate

$$\int_0^{\infty} \frac{\cos \lambda x}{x^2 + a^2} dx.$$

- (b) An infinitely long string having one end at $x=0$, is initially at rest along the x -axis. The end $x=0$ is given a transverse displacement $f(t)$, $t > 0$. Find the displacement of any point of the string at any time.

Unit-II

3. (a) If $\cosh x = \sec \theta$, prove that $x = \log \left[\tan \left(\frac{\pi}{4} - \frac{\theta}{2} \right) \right]$ and

$$\tanh^2 \frac{x}{2} = \tan^2 \frac{\theta}{2}.$$

(b) If $\sin^{-1}(u + iv) = \alpha + i\beta$, prove that $\sin^2 \alpha \cosh^2 \beta$ are the roots of the equation $x^2 + (1 + u^2 + v^2)x + u^2 = 0$.

4. (a) An electrostatic field in the xy -plane is given by the potential function $\phi = 3x^2y - y^3$. Find the stream function.
- (b) Find the bilinear transformation which maps the points $z = 1, i, -1$ into the points $w = 2, i, -2$ respectively. Find the fixed and critical points of the transformation.

Unit-III

5. (a) A man known to speak truth 3 out of 4 times. He throws a die and reports that it is a six. Find the probability that it is actually a six.

(b) The frequency function of a continuous random variable is given by $f(x) = y_x \cdot x(2-x)$, $0 \leq x \leq 2$. Find the value of y_x , mean and variance of x .

6. (a) The probability that a bomb dropped from a plane will strike the target is 0.2, if six bombs are dropped, find the probability that:

- (i) exactly two will strike the target,
- (ii) at least two will strike the target.

(b) Students of a class were given a mechanical aptitude test. Their marks were found to be normally distributed with mean 60 and standard deviation 5. What percent of students scored:

- (a) more than 60 marks?
- (b) less than 56 marks?
- (c) between 45 and 65 marks?

Unit-IV

7. (a) Define:

- (i) Feasible solution
- (ii) Basic solution
- (iii) Basic feasible solution
- (iv) Non degenerate BFS
- (v) Degenerate BFS
- (vi) Optimum Basic feasible solution
- (vii) Unbounded solution.

(b) Using graphical method, solve the LPP

$$\text{Maximize } z = 7x + 3y$$

subject to

$$x + 2y \geq 3,$$

$$x + y \leq 4,$$

$$0 \leq x \leq 5/2,$$

$$0 \leq y \leq 3/2$$

8. Solve the following problem using simplex method.

$$\text{Maximize } Z = 7x + y + 2z$$

subject to

$$x + y - 2z \leq 10,$$

$$4x + y + z \leq 20, \text{ and}$$

$$x, y, z \geq 0$$