

CSE
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X IT
Roll No.

Total No. of Pages : 4

BT-3/D06

8497

DISCRETE STRUCTURES

Paper : CSE-205E

Time : Three Hours]

[Maximum Marks : 100

Note :— Attempt any FIVE questions.

1. (a) Show that $z^n > n^3$ for all $n \geq 10$.
- (b) When n couples arrived at a party, they were greeted by the host and the hostess at the door. After rounds of handshaking, the host asked the guests as well as his wife to indicate the number of hands each of them had shaken. He got $2n + 1$ different answers. Given that no one shook hands with his or her spouse, find the number of hands the hostess had shaken.
- (c) Among 50 students in a class, 26 got an A in the first examination and 21 got an A in the second examination. If 17 students did not get an A in either examination, find the number of students who got an A in both examinations.
- (a) A chess player prepares for a championship match by playing some practice games in 77 days. She plays at least one game a day but not more than 132 games altogether. No matter how she scheduled the games, prove that there is a period of consecutive days within which she played exactly 21 games.
- (b) Show that the transitive closure of a symmetric relation is symmetric. Is the transitive closure of an antisymmetric relation always antisymmetric ? Justify.
- (c) A man hiked for 10 hours and covered a total distance of

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(Contd.)

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45 kms. He hiked 6 kms in the first hour and only 3 kms in the last hour. Prove that he must have hiked at least 9 kms within certain period of two consecutive hours.

3. (a) Construct the truth table of the proposition

$$(p \rightarrow (q \rightarrow r)) \rightarrow ((p \rightarrow q) \rightarrow (p \rightarrow r)).$$

- (b) Given that the value of $p \rightarrow q$ is false, determine the value of $(\bar{p} \vee \bar{q}) \rightarrow q$.

- (c) Among $3n + 1$ objects, n of them are identical. Find the number of ways to select n objects out of these $3n + 1$ objects.

4. (a) Find a particular solution of the recurrence relation

$$a_r + 5a_{r-1} + 6a_{r-2} = 3r^2 - 2r + 1.$$

- (b) Find the generating function of the difference equation

$$a_r - 5a_{r-1} + 6a_{r-2} = 2^r + r, \quad r \geq 2,$$

with the boundary conditions $a_0 = 1, a_1 = 1$ and hence solve it.

5. (a) Define a

(i) semigroup;

(ii) monoid;

(iii) group.

Give an example of

(i) a semigroup which is not a monoid;

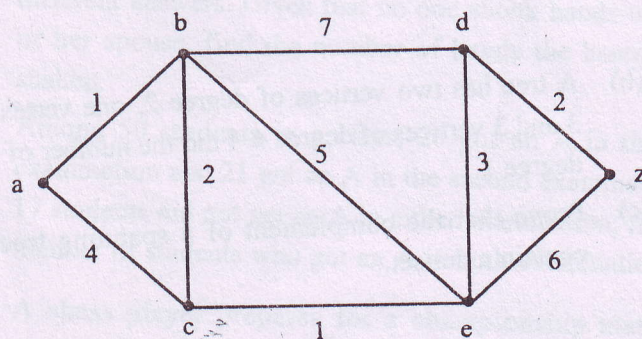
(ii) a monoid which is not a group.

- (b) Let G be a group and $f : G \rightarrow G$ a map given by $f(a) = a^{-1}$, $a \in G$. If f is a homomorphism, prove that G is Abelian.

- (c) Given an integer $n \geq 2$, prove that the set of all n^{th} roots of unity is a cyclic group of order n .

6. (a) If H is a subgroup of a group G , prove that the left cosets aH, bH of H in G are either disjoint or are identical.
- (b) Let $m \geq 2$ be an integer. Prove that the set Z_m of integers $\{0, 1, 2, \dots, m-1\}$ with addition and multiplication modulo m is an integral domain if and only if m is a prime. If m is a prime, is Z_m a field? Justify.
7. (a) What do you understand by
- a simple path;
 - an elementary path; in a graph G ?

Describe a procedure to find shortest path from a given vertex a to any vertex in G and use it to find a shortest path from a to z in the graph.



- (b) For any connected planar graph, prove that
- $$v - e + r = 2$$
- where v, e, r are the number of vertices, edges and regions of the graph respectively.
8. (a) A tree has $2n$ vertices of degree 1, $3n$ vertices of degree 2 and n vertices of degree 3. Determine the number of vertices and edges in the tree.

- (b) A tree has two vertices of degree 2, one vertex of degree 3 and 3 vertices of degree 4. Find the number of vertices of degree 1.
- (c) Prove that the complement of a spanning tree does not contain a cut-set.