Roll No. ....

Total No. of Pages

BT-3/J05 **85**

DISCRETE STRUCTURES

(Common with CO., IT)

Paper : CSE-205 E

(According to Syllabus Dec. 2004)

Time : Three Hours] [Maximum Marks :

Note :— Attempt any **FIVE** questions.

1. (a) By mathematical induction, prove

$$1 + a + a^2 + \ldots\ldots a^n = \frac{1 - a^{n+1}}{1 - a}.$$

(b) Prove principle of Inclusion and Exclusion for n sets.

2. (a) Construct truth table for :

(i) $(p \to p) \vee (p \to \overline{p})$

(ii) $(p \to (q \to r)) \to (p \to q) \to (q \to r)$

(iii) $p \leftrightarrow (\overline{p} \vee \overline{q})$.

p, q, r are propositions. 10½=3½

(b) Formulate the Tower of Hanoi problem as a Recurrer relation and then solve this recurrence relation.

4+5½=

3. (a) Clearly define Lattice, Antichain.

(b) Let R be a binary relation on set of all strings of 0s and such that R = {(a, b) | a, b are strings that have same num of 0s}.

Is R reflexive ? Symmetric/Antisymmetric ? transitive ? it equivalence relation or p.o. relation ?

(c) From integers 1....200, 101 of them are chosen arbitrari

How many minimum pairs are found to exist such that one divides another in that pair ? 6

4. (a) Using generating functions, solve

$$a_r - 2a_{r-1} + 2a_{r-2} - a_{r-3} = 0$$

given that $a_0 = 2$, $a_1 = 1$, $a_2 = 1$. 12

(b) Three integers are selected from integers 1, 2, ..., 1000. In how many ways can these integers be selected such that their sum is divisible by 4 ? 8

5. (a) Let (A, $*$) be a semigroup where

$a * a = b$. Show that :

(i) $a * b = b * a$

(ii) $b * b = b$. $4\frac{1}{2}\times2=9$

(b) Show that an integral domain that has a finite number of elements is a field. 11

6. (a) Let (F, +, ·) be the field of integers modulo 2 & ($F[x]$, $\boxed{+}$, $\boxed{\cdot}$) be corresponding ring of polynomials. Construct the ring of polynomials modulo $1 + x + x^2$ 12

(b) State Lagrange's theorem and give some example of it. 8

7. (a) Prove clearly necessary and sufficient condition for Eulcarian path. 11

(b) n cities are connected by a network of K highways. Show that if $K > \frac{1}{2}(n-1)(n-2)$, then one can always travel between any two cities through connecting highways. 9

8. (a) Prove that, by coalescing of vertices, a minimum spanning tree can be obtained from a graph. 8

(b) A tree has n_2 vertices of degree 2 n_3 vertic
3 & ... n_k vertices of K degree. How many verti
1 does it have ?

(c) How many edges are common in every circu
cut-set ? Justify your answer.